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T H E R A P I D R E S U L T S C O L L E G E .

ECONO4ICS AND THE
TOOLS YOU NEED

This set of lectures contains an introductory overview of the subject together with a revision of the basic "tools" you will need to make your study of Economics so much easier. It comprises:-

RRC 15 391

A.

ECONOMICS IN PERSPECTIVE.

APPROACH TO THE SUBJECT

Because Economics is probably an entirely new subject to you, I should like, right at the outset, to have a few words with you on your approach to the subject.

In some respects Economics resembles Law; just as in the latter subject you have to do two things --- (l) memorise the principles of Statute and Common Law, and (2) learn how to apply these principles to cases taken from everyday life --- so in Economics you must (1) memorise the Economic laws or principles which have been enunciated by the past masters of the subject, and (2) learn how to agely these Brincigles to cases taken from everyday life.

To assist you in developing this necessary power of applying principles to practice, a number of problem questions have been included at the end of each lecture. You should first attempt these questions yourself and then compare them with the answers given. The conclusions you have drawn may be different from those given in the model; that does not matter. The essential point is: is your reasoning Logical? Similarly, your "judgment" on a case given in the examination paper may be different from that which would be given by the examiner. Never mind! If you have supported your judgment by sound reasoning, the examiner should give you good marks for your answer. But be careful not to over-emphasise your personal opinions. The primary object of the examiner is to see whether you have studied Economics as a subject, and he will not be assisted in this object if you swamp your learning with your own views on a particular point.

WHAT IS ECONOMICS?

There is no absolute certainty about the subject we are going to study. This is not surprising, really, because opinions on the meaning and scope of economics have tended to change as the science has developed.

For all practical purposes, it is sufficient to note that Economics began to be formulated as a subject in the Seventeenth Century; but no outstanding attempt was made to define the scope of the subject until Alfred Marshall wrote his "Principles of Economics". Before the publication of this book in 1890, the subject we are discussing had always been called "Political Economy".

Marshall’s definition:

Marshall’s definition of the subject was,
"Economics is the study of man’s actions in the ordinary business of life; it enquires how he gets his income and how he uses it. Thus it is on the one side a study of wealth and on the other and more important side, a part of the study of man".
In other words, Marshall considered that Economics was an analysis of how man acquires his earnings and how he disposes of them.

The Modern definition:

The basic substance of Marshall’s definition is still accepted, but more modern economists have introduced refinements into it. Whereas Marshall resorted to money costs, on the grounds that there was no suitable way of expressing real costs, modern economists have devised a method of describing real costs in terms of sacrificed alternatives. (Sometimes referred to as the concept of oegortunitx-cost.) Money spent on a fur coat cannot be spent on entertainment as well. Similarly, time spent in working overtime cannot be spent in leisure; the more a man indulges in one of these alternatives, the less he can indulge in the other,
Man’s wants are limitless; when he has satisfied all his material wants, he develops cultural desires, such as poetry, music and literature. The means available to satisfy his wants are limited, however, so that man is faced with alternative uses of available resources. From this fact the modeIn definition of Economics is derived; in the words of

Professor Robbins, "Economics is concerned with the economising of scarce means in the attaining of given ends". The importance of scarcity must be remembered. From the point of view of Economics, anxthing that commands a Brice, however low, is scarce. Pa ment of a rice indicates readiness to make an effort or a sacrificeZ and this instantlx gives rise to economic considerations. Moreover, the absence of a recognised price for a commodity does not mean that that commodity cannot be ranked as on economic good. Thus, water may be an economic good even though a man has been in the practice of drawing his supplies from a nearby stream; to fetch the water himself involves the use of time which could have been spent in other ways, and if another person is employed to fetch the water for him, this other person would have to be paid for his service, while if a pumping plant is installed, its construction would involve the outlay of capital which could have been used for something else.

Professor_McConnell defines economics as:

"The social science concerned with the problem of using or administering scarce resources (the means of producing) so as to attain the greatest or maximum fulfilment of society’s unlimited wants (goals of producing)."

Therefore to summarise:

Whatever the definition of economics presented, whether by socialists or capitalists or those in between, it usually contains the following components:-

1. Fixed Resources,
2. for which there are unlimited uses;
3. A value system exists based on prices and markets or government decree;
4. Such resources need allocating;

5. Goods and services must be eroduced, distributed and exchanged and most importantly, an economist working under any system would ask;

6. How efficiently are these things being done? What do we get out compared with what we put in: as a percent indicating Broductivitx.

C. ECONOMICS AS A SOCIAL SCIENCE

Economics is one of the social sciences that studies man in relation to his economic environment and its forces. Just as Ethics which deals with man’s conduct, Psychology which deals with his mental experiences, Law which deals with his rights and duties, are all social sciences, so Economics is a social science concerned with man’s material welfare and with his conduct in securing it.

output

input

None of these divisions of the social sciences is quite distinct, for every science is related to other departments of knowledge and all affect man in the ordinary business of life. Each has to trespass to some extent on the preserves of the others, and this is particularly true of Economics. The effect of economic forces is being continually controlled and modified by social forces --- the customs and laws of society and its ethical and moral standards.

Frequently, during the course of these lectures, I shall have occasion to enunciate and explain what are known as economic laws,which are statements of principles underlying man’s conduct in the ordinary business of life. At the very outset, however, I want you to realise that most of these laws are merely statements of tendencies that, given certain conditions, will be followed by certain results.

Economic theory is not an exact science like Mathematics. Because we have to apply the theory to solve real life problems and develop workable solutions it is difficult to be exact. We are dealing with human beings, not test-tubes as in laboratory experiments.

Many economic laws have a strictly limited application, and are liable to be considerably modified by disturbing influences. From this it does not follow, of course, that such economic laws are basically untrue or without practical value. They are useful statements of general tendencies, and serve as a starting point for further investigation.

For example on page 15 of "Positive Economics" Professor R. Lipsey outlines in diagrammatic form the method used by economic theorists as follows:

DEFINITIONS
AND
HYPOTHESES ABOUT BEHAVIOUR
(often called assumptions)
process of
logical deduction
PREDICTIONS
(often called implications)
CONCLUSION:
that the theory is either
REFUTED BY
or
CONSISTENT
WITH THE FACTS

D. THE SCOPE OF ECONOMICS

The economics principles and laws that we shall study are those put forward by economists who have analysed the influences and results of tendencies which operate in everyday life.

For example, the product of the land varies from year to year; farmers have lean seasons and prosperous seasons. He read of farmers complaining that when their crops are good, prices are so low that the crops are not worth harvesting. Economic principles and theories set out to explain why these things should be; why the price system functions as it does.

We have already seen that Economics is concerned with man’s material existence. All of us have needs which must be satisfied if existence is to continue. In advanced communities these primary needs can be met with greater ease. By co-operating with his fellowemen and by using technical equipment to aid him in his labour, man has succeeded in producing more than sufficient to satisfy his mere animal needs. But his wants have become many and varied and a highly involved system has gradually been built up to supply him with these, in the face, always, of limited resources. This is the scope of Economics --- to consider how man’s wants are satisfied.

We can build three models to illustrate the role of different sectors in the satisfaction of man’s wants. The first two diagrams are from Heilbronner and Thurrow’s "The Economic Problem".

The Two Sector Model
HOUSEHOLDS
MONEY SERVICES OF
PAYMENT LABOUR, LAND,
FOR GOODS AND CAPITAL
AND SERVICES PRODUCED BY
HOUSEHOLDS
GOODS AND MONEY
SERVICES PAYMENT
PRODUCED FOR LABOUR,
BY BUSINESS LAND AND
I CAPITAL
BUSINESSES
The basic market mechanism with only two sectors
illustrated : households and businesses.

JUNMHL; m H 19m I ly1_._vm_(_,_z, 23.
Thlvc Sector Modol
i.c. with government included.
The Four-scctor Model includes households,
government and trade. It illustr
and services created by resource
businesses,
ates the flows of goods
5 and paid for by money.

NOTE:
ECONOMICS IN PERSPECTIVE. PAGE 9.
SAVINGS (-)
INVESTMENT ( )
Savings are treated as a leakage from, and investments treated as an injection to, the flow.

E. THE ECONOMIC PROBLEM

l.
TYPES OF ECONOMIC SYSTEMS:

Largely through geographical factors, man has tended to-become grouped into great communities, politically described as notions. By virtue of their independent development, the economic systems in these various communities often differ in a number of respects. Different communities tackle in different ways the economic problem of adapting the available resources, both human and material, to meet the wants of the people who make Up the community. The two extremes are central Blanning and laisseZefaire, (non-interference) sometimes referred to as state control and private enterprise, or as socialism and capitalism. Between them lies a mixed system, which most West-European economies have become since the last World War.

(a)
Central Elanning:

This is also known as the command economy. All decisions are taken by a central planning authority. The decision-making process moves from central planning boards to enterprises that are either state-owned or regulated. Consumer sovereignty largely gives way to the collective preferences of the central planners. Decisions are taken on the following:-

(i) production targets
(ii) allocation of resources
(iii) distribution of income
(iv) desired rate of growth.

The state owns the means of production and most property. It also determines the incomes for various levels of labour.

It cannot operate succesfully unless it has very extensive powers. It cannot decide upon a distribution of the coal output unless it can foretell how much coal will be produced, and it cannot predict how much coal will be produced unless it has the power to order men to work in the coal mines; i.e., the planning authority must have powers which enable it to "direct" labour.

This task presents a very formidable problem:

(i) There must be an organisational chain of command
(ii) There is u serious problem of coeordination arising From the interdependent nature of the modern industrial economy. The production of one sector depends upon the outputs of another sector.

(iii) There is a problem in maintaining efficient allocation of scarce resources.
(iv) What do the central planners rely on to set goals if consumer preferences are not taken into account?
Soviet Russia uses this type of command economy which we call Communism.
Laissez-faire:
The laissez-faire or market economy works on the assumption that human beings behave in such a way as to achieve maximum money gains. All production is for sale on the market and all incomes derive from such sales. By the system of laissez-faire, each member of the community sets about solving his own individual problem. Nobody guides the economy as a whole - everything is decentralised into millions of individual decisions taken by consumers and producers. Complete laissez-faire is never found in practice, however, for it would result in anarchy. There must be some supreme authority to prevent people seizing the fruits of the efforts of their fellow-men whether by fraud or by force.In most cases, too, it is found advisable to institute private property rights (i.e., the rights of persons or bodies to claim that certain assets belong to them for use in any way they think fit, provided it is not detrimental to the interests of the community at large.)
Between these two extremes there are a variety of hybrid (or complex) systems: there may be central planning together with some degree of capitalism, but more often there will be private enterprise combined with a small or large measure of state interference.

A Polish economist named Lange worked out a combination of command and market economies whereby a state of market socialsim exists. The state will own the means of production except that labour will continue to control its own services and will be free to offer these services on the market for wages. The State also subsidises the labourers in order to maintain a more equal distribution of income. Consumers may purchase what they wish but the State will determine the proportion of income devoted to capital accumulation. Firms are instructed to behave as they would under conditions of perfect competition.

EFFICIENCY OF THE ECONOMIC SYSTEM:

The most efficient system is that which makes the most productive use of the limited human and material resources at its command.

An efficient system produces the maximum amount of necessary and desirable goods and services with the minimum of human effort and sacrifice, and produces them too, in the Broeortion they are wanted by the community. The system will have reached its maximum efficiency "if it is impossible to increase satisfaction by producing more of one good and less of another". Here, of course, we are again faced with the conception of sacrificed alternatives.

In the realm of economic policy, the individual will look to the following aims:

(a) Emeloyment:

The Government should aim at maintaining a high level of employment. The term usually Used is "full employment", which is taken to mean that there are more jobs available than there are unemployed to fill them. But this is an over-sim
must not be taken literally. Always, in every country, there exists a relatively large group of permanently unemployed people - the too old, the too young, the incapable, the drop-outs. Thus
essentially a relative term.
plification; it
"full employment" is

(b)

If private industry is unable to offer employment to work-seekers, some of the unemployed at least may be absorbed into Government undertakings; a rational "public works policy" is an e55ential part of a "full employment policy". What the man in the street is more concerned with, however, is that the Government should create conditions which will give a high and stable level of employment, and that it should at all costs avoid a serious and prolonged depression of trade.

An imgroved standard of living and the reduction of economic inegualitx:

The desire for improved standards of living is actually a desire for increased real income, i.e., more material possessions.

Real income refers to purchasing power in terms of goods that can be bought: if money income is constant and prices rise, real income obviously drops as less goods can be purchased for the same amount of money. Similarly if prices fall, real income rises.

As standards of living rise, greater amounts of consumption goods (e.g., bread, clothing, etc.) will be consumed per head of population. The role of government is to make this possible while, at the same time, ensuring that sufficient resources are devoted to producing capital goods to ensure increasing levels of consumption in the future.

Related to this concept is the problem of economic inequality, i.e., poor income distribution.

The standard of living in a country is usually measured in terms of the average income per head of population (per capita). Obviously, if the wealth is concentrated in the_hands of a few (e.g., Arab oil states), the average standard of living may be very low despite high average per capita incomes.

On the other hand, complete equality would rob efficient workers of much of their incentive and, in a private enterprise economy, would reduce capital available for industry as saving and investment would be reduced.

The balance settled upon is dictated by the politics of the party in power and the opinions of the business community.

Imgroved working conditions:

The conditions under which people work have an important bearing on their general welfare and happiness.

Although it is those who provide the capital for industry (i.e., the investors or shareholders) who usually have the largest say in the management and general policy of an industry, the Government introduces certain regulations to prevent the exploitation of workers by unscrupulous employers, e.g., maximum hours of work and minimum conditions of safety and hygiene, etc. The workers themselves may exercise a voice through trade unions, works management committees and other similar organisations.

Social securitx:

It is generally believed that it is the duty of every State to provide a minimum standard of economic welfare for all its citizens, and that the State should provide, therefore, for citizens who have been afflicted by some hardship such as inability to find employment, or inability to work, for physical reasons. The lengths to which the provision of such "social services" should go is a matter of economic policy.

In "Economics", Benham discusses four questions of principle which arise in connection with the provision of social security.

(i) All persons who have lived in the country For a specified minimum period should be eligible for benefits, whether they are normally woge-eorners or not.

(ii) If the benefits are to achieve their objective it may be desirable to pay some wholly or partly in kind, or to exercise some control over the manner in which money benefits are spent.

(iii) To achieve their purpose, benefits must not be too low, but at the some time if benefits are too high they may reduce incentive to find employment.

(iv) The sharing of the cost by workers, employers and taxpayers may give the idea of insurance, but unless the rates of contribution are graded in some way to allow for differences in risk (e.g., with respect to type of employment, age of worker, standard of fitness, etc.) it is really only a special form of taxation.

(e) A stable currency:

To promote economic stability, to avoid depreciation in the value of savings, and to ensure that those who have provided a fixed annuity or pension for their retirement do not find that in terms of goods their incomes are gradually falling in value, one aim of economic policy should be a stable unit of currency. However, a Government policy designed to prevent prices from rising may lead to a falling off in the level of employment. This conflict and others will be dealt with in greater detail later on.

F. tSTEPS IN ECONOMIC ANALYSISl

The traditional approach to economic analysis was to consider first production, then distribution, and then consumption or exchange.In practice, of course, production and consumption act concurrently.

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Mndmn OLDMUIIIl’HS huvu lnlmn qlmvf pains to emphasiso this puinl; lhny havo formulated u syslnm known as oguilibrium nnulysin in which account is taken of all the various foclors whirh muy influencn un nrnnnmic detision. These factors will HIM) im lmle (:mlnin (hunger, Wllit’h might conceivably affect lho (lnlJKlOVI (1! Hit! lust minute.

r0! u syslnmuilg scientific investigation it is best to study lhn nifnrlw nf LhHHQOS in isolation. To study changes in lsnlntion, i1 is necessary to make the assumption "ceteris purihus" --- all things being equal or held constant while we examine them in turn. For example, one might consider the various things ("determinants") which determine the price of an article ("a (ood", in Economics); the tastes of the consumers also play a part. In this particular example, in order to analyse the effect of price changes on the demand for the article (or "good"), if is assumed that the tastes of the consumers remain constant, and that, all things being equal, if the price rises the demand will diminish. Using the language of Economics, one would say, "ceteris Boribus, if prices rise, demand will decrease".

l. PRGRKZTIW:

lo satisfy his wants, man has to utilise to the best advantage the natural resources and the products around him. This is the sphere of production.

Anything which is used by man (including his own labour) in furthering production, is said to be a factor of Eroduction. Clearly there is a very great variety of such factors in daily use, but in economic discussions

(0) Factors which cannot be bought and sold by men (9.9., sunshine) are usually excluded.

(b) lhe other factors of Broduction are:-

Lnnd - which earns rent

Labour _ which earns wages and salaries

Capital - which yields interest
Organisation - or the "entrepreneur" who earns profits.

DISTRIBUTION:

Every factor which contributes to production is naturally entitled to a share in the product. Obviously, it would be difficult to apply this procedure in cases where the product was indivisible.

Division of the product is much simpler where money is used. The finished product is sold to consumers and the proceeds are distributed as money rewards to the contributing factors. Thus, labour earns wages, land earns rent, capital earns interest and entrepreneurship earns profits.

Under Distribution then, we study what determines these various monetary rewards.

CONSUMPTION:

Production is undertaken to meet the requirements of consumers. These requirements are indicated by the willingness of consumers to pay a price. On account of the technological structure of industry, however, some considerable time must now-o-doys elapse between the beginning of production and the marketing of the finished product, so that producers must onticigote the requirements of consumers. Moreover, the ability of consumers to pay prices depends upon the amount of income they derive from their contributions to production. In other words, money and technical progress have brought an economic circuit into being; Production makes possible the payment of incomes, and these incomes in turn make it possible for the recipients to buy the results of prpduction, while exchange or consumption (via the money system) permits convenient transfer of title to goods and services. i

G.

ECONOMIC MAN

An important assumption which an economist must make if he is to be able to deduce reliable principles regarding behaviour, is that all men are "economic" men; i.e., that men are primarily interested in promoting and satisfying their own wants.

Critics of the science often seize upon this point to contest its usefulness in real life, but you will agree, I think, that by adopting as his "specimen" a man who is assumed, when he is faced with a choice, always to adopt the course which is more favourable to himself, the economist is not deviating far from reality. Similarly lawyers talk of the "reasonable man".

The economist, in making this assumption, is attributing to man the characteristic of rational behaviour and rational choice. This consistent behaviour is a necessary basis for the development of sound economic theory.

H. NORMATIVE AND POSITIVE ECONOMICS

Positive Economics deal with "
deals with what "ought to be".
what is"; Normative Economics

Positive Economics is independent of any particular ethical position. "Its task is to provide a system of generalisations that can be used to make correct predictions about the consequences of any change in circumstances". (Friedman)

Normative Economics cannot be independent of Positive Economics. Policy decisions (judgements are the basis of Normative Economics) necessarily rest on predictions which must be based on Positive Economics.

Thus while the statement that one production method is more economical than another may be positive, the decision on what should be produced is normative.

To any government which is really concerned with the well-being of the country (as opposed to its own well-being) there are always certain aspects of the country's economy which, it feels, should be sensitive to, if not actually under, government control or influence. These views of responsibility are normative or VALUE JUDGEMENTS, and no unequivocal presentation of cold, hard fact will be likely to sort out differences of opinion without controversy, because the opinions of men are always bent, to a greater or lesser degree, by personal principles or pet philosophies: infinite argument is possible as to what could happen if this or that step were taken, or this or that policy were adapted.

1. METHODS OF STUDY IN ECONOMICS

1. INDUCTIVE AND DEDUCTIVE METHODS

All economists do not use the some methods. Some collect statistical or other data From the world around them and by analysing this data, extract laws relating to tendencies. They use what is known as the Inductive Method of Study. Others start off with certain assumptions and by deductive reasoning arrive at the conclusion that if their assumptions are valid, then certain results must follow. These economists use what is known as the Deductive Method. Although neither of these methods can be condemned out of hand, there is no doubt that the most satisfactory method for an economist who wishes to arrive at conclusions which are both realistic (i.e., of some use in the everyday world) and theoretically unchallengeable, is one which combines inductive research with deductive reasoning.

2. THE USE OF MATHEMATICS

Economists also have different ways of setting out their arguments. Some rely entirely on their literary powers while others place great dependence on mathematical explanations. Marshall, one of the leading economists of the Nineteenth Century, used both. He first deduced a law by mathematical methods; he then translated his mathematical theorem into prose; thirdly, he delved into the business and commercial world until he had found some data from the real world which substantiated his theory; then, and only then, did he give his theory final shape for presentation to the public. In Marshall’s case all algebraical and geometrical arguments were relegated to footnotes or appendices in his "Principles of Economics", but other economists such as Keynes or Joan Robinson make extensive use of mathematical methods in the main sections of their published work.

In this Course, and in your examination work, you will rely primarily on written explanations. Unless you have reached a high standard in advanced mathematics you are likely to find that algebraical methods tend to obscure rather than clarify the issue. With geometrical diagrams, however, the position is different. Only a limited knowledge of graphs will enable you to understand (and present to the examiner) illustrations which make it possible clearly to explain in a few lines what would otherwise take a page or so.

ECONOMICS IN PERSPECTIVE

There is no definiteness about the scope of Economics. Marshall’s definition was widely accepted until modern economists improved upon it by emphasising the scarcity aspect of man’s resources. But there is general acceptance that it is a science (though not an exact one) and is a branch of sociology. The primary object of economic activity is to provide man with his basic needs --- food, clothing and shelter; but technical advances have made it usual to provide for cultural wants in addition to these basic needs. The efficiency of on eeonomic system is judged by the volume of satisfaction it achieves from the available economic resources. It is not for the economist to express an opinion on the desirability of central planning or loissez-foire, or on any system using some of the features of both; the economist con merely point out the advantages and weaknesses of each in their efficiency as economic systems.

In the real world, production, consumption and distribution are, broadly speaking, concurrent or even simultaneous processes, but for the purpose of economic analysis we study each of these phases in isolation, although by using the principle of "ceteris paribus" we may achieve a degree of exactness ot the expense of reality.

The concept of economic man, a man of selfish interests, is used here for purposes of scientific enquiry.

PROGRESS QUESTIONS

Set out at the end of each lecture you will find two or three questions bearing on the material you have just studied. These questions represent the type of questions that will be asked in the final examination. It is not intended that you should send in your answers to these questions. They are included, in order

1. to stimulate your thinking capacity, and
2. to enable you to evaluate yourself and the thoroughness, (or otherwise) of your study.

The main points which should be included in your answer are sometimes appended - in note form - to each lecture: from these you can correct your Own answers.

You should take particular core in answering these questions, and you can rest assured that if you can handle competently all those set on all the lectures, you will have nothing to fear in the examination itself.
It is left to your own judgment whether you actually write a specimen answer to each question, or whether you merely turn over in your mind the points that you would make, but whichever method you employ, tackle each question conscientiously. If you adopt the latter method, give yourself, from time to time, the sterner test of writing your answers.
I would like to emphasise that should there be some point which is not clear in your mind, do not delay in writing to me about it. Explanation and advice are offered freely, and you must not hesitate to avail yourself of this service.
Answers to the Test papers which you will find at the back of each lecture booklets must be written, and submitted for marking. These answers must be written in essay form - the form in which the Final examination must be done.
At the head of each of these Tests you will find a stipulation as to how much TIME you are allowed for the completion of the Test. Be careful to stick strictly to the given time limit; don’t "cheat", and give yourself extra time to complete the last answer - the invigilator in the examination room will not give you one single minute more than the stipulated time!
If you need more time to cover any Test, it is clear that either your thinking, or your writing, or BOTH, need to be speeded up.
Here are the typical questions based on this lecture:-
1. What is the scope of Economics? Examine its relationship to other social sciences.
2. What claims has Economics to be regarded as a science? Consider its relationship to other sciences and explain why economic "laws" are less exact than the laws of the physical sciences.
3. "A study of society’s economic problems is basically an enquiry into problems connected with scarcity, choice and exchange." Discussh

GUIDES TO THE ANSWERS

QUESTION 1:

Economics is one of the social.sciences which together make up Sociology. Just as Ethics deals with man’s conduct, Politics with his position in the State and Law with his rights and duties, so Economics is concerned with man’s material welfare; it is another social science, the sphere of which is man’s conduct in the ordinary business of life.

None of the social sciences is an entirely separate subject which can be put into a woter-tight compartment. Every science is related in some way to other departments of knowledge, and all affect man’s everyday existence in some way or another. Moreover, each has to trespass to some extent on the preserves of the others, and this is particularly true of Economics. Economics cannot be kept entirely distinct from Politics, Law and Ethics. The effect of economic forces is being continually controlled and modified by social forces --- the customs and. lows of society and those ethical and moral standards to which society must conform.

"Economics represents an attempt to systematise or organise our knowledge of human society in its efforts to satisfy its needs" (Jones). It analyses the relative advantages of different systems of government from the point of view of this motive. Since Economics deals mainly with human forces and not, like Physics for example, with natural forces, it cannot be said to be on exact science; to achieve exactness assumptions must be made, and the economist must deal with the average man, or with groups of people, thus eliminating the eccentricities of individuals.

QUESTION 2:

Scientific generolisations refer to reality; economic generolisotions also refer to reality, even though they lack the exoctitude of some, but not all, other sciences. For example, the Law of Diminishing Marginal Utility, though real, cannot be stated with the exactitude with which the Low of Gravity can be expounded.

Furthermore, economic science deals with man and his material welfare; though he cannot be accurately defined in a scientific sense, he does exist, and for the purpose of Economics on "economic man" has been propounded --- one influenced by selfish motives. While this device may cause Economics to lack a certain amount of exactitude, it does not, merely because of it, cease to be a science. In Economics 05 in other social sciences like Psychology, Politics and Ethics one is dealing with human reactions and the psychological aspect onnot be ignored. However, these reactions do tend towards a fundamental similarity, therefore it is possible to make assumptions and generolisotions about behaviour.
The methods of economic reasoning adopted are often similar to, but may be very different from, the methods adopted by the exact sciences. The inductive and deductive methods both form part of the reasoning of Economics.
In the deductive method, certain accepted assumptions, or hypotheses, are used as a starting point from which to proceed to other propositions.
In the inductive method, facts are collected, collated and classified, and from this a generalisation is made.
Though Economics has some affinity with the exact sciences, it is more properly regarded as 0 social science, because it is concerned with a certain and limited part of man’s activity. Just as Law is concerned with man’s rights and duties, Ethics with his conduct, Psychology with his behaviour, so Economics is concerned with "his ordinary business of life" --- this being Marshall’s definition. That is, Economics is concerned with a section of man’s activity and it is therefore a social science. Naturally, there is bound to be a certain amount of overlapping with other social sciences. For example, the vendor of polluted food is indulging in economic and also unlawful conduct.
If, owing to its comparative youth, the science of Economics has not as yet been defined in a "final" or exclusive sense, it is still no less a science. For after 011, Economic Lows must first of all be discovered and applied before the scope of the science reveals itself for the expounding of precise definition.

ECONOMICS IN PERSPECTIVE. PAGE 25.

QUESTION 3:

If resources were unlimited there would be no economic goods and no economic activity as we know it. If there were enough of everything, or enough resources to make it possible to produce as much of everything as would fully satisfy the wants of everybody, there would be no economic problem. It would not matter if too much of a particular article was produced; it would not matter if labour and machinery were combined in a ratio which did not happen to be the most efficient. But this is not so.

In the world as it is goods and resources are scarce. Our problem is to make the best use of the limited means that we have and this makes it necessary constantly to choose between alternative uses of what goods we do have and, in some cases, of the time at our disposal. This is sometimes referred to as the low of scarcity. The real economic cost of producing a certain article is often said to be the product which could have been produced it the resources were used in some alternative way. If for some reason we are unable to use the resources in the alternative way and we require the alternative product which could be produced, this may be obtained by exchange. In other words, exchange is an indirect method of making a choice. Clearly if goods were in unlimited supply there would be no need to give something in exchange for whatever it is we wont.

RRC. 14 028

Wt. -.
n m, m.

A WORD BETWEEN LECTURES.

Having completed the first lecture in your course you will, I"m sure, agree that Economics is not the difficult subject everyone thinks it is. It deals, after all, with ordinary every-doy things which we can all easily relate to!

Youull recall what we said at the end of the first lecture: You don"t need advanced mathematics in order to understand Economicsi but you g2 need some knowledge of graphical presentation of date. You also need some BASIC arithmetic and mathematics in order to appreciate more fully the graphical and other concepts and logical arguments found in your course.

Many past students have said to me "If only my basic maths had been stronger I would have found the study of Economics so chh easier - almost effortless!" With this in mind, your tutor has selected a number of BASIC mathematical "tools" as a small refresher course to help you.

Naturally, if you know that you are strong in Maths, feel free to skip the rest of this set. If you have doubts about your abilities, please study the following lectures with care before proceeding.

Enjoy your studies, and keep to your time-table!

DIRECTOR OF STUDIES.

RRC 15 392

:11
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l. 'Ihwml q
W 1-5.. (fly: IWIIrmgtu
?'zr.. . - .xm-IFI um
II.IH'l 4-..I'I'
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F R A C T I 0 N S a

lDECIMAL FRACTIONS:I

A decimal fraction is a fraction in which the denominator (usually omitted) is 10 or some power of 10, i.e. 100, l 000 and so on.

In any number such as 543,672 it can be seen that the 3 stands for 3 unitsy the 4 for 4 tens and the 5 for 5 hundreds, so that each figure to the left increases tenfold at each step from the units figure. That is, the 5 which is two steps from the 3 stands for 5 x 10 x 10 or 5 hundreds. Similarly then, when proceeding to the right each figure is decreased tenfold. Thus the 6 which is one place to the right of the units figure
represents I3 ; 7, which, is two places to the right of the units
fi ure re resents --Z-- or -Z_- while the 2 re resents
9 ’ 9 10x10 100’ P
2 2
10 x 10 x 10 or 1000

Therefore 543,672 may be written in full as:
5x100t4xlOt3t-6-k-Z- 2
10 100 " 1000’

The comma is called the decimal comma while such fractions are called decimal fractions or more shortly decimals. The places to the right of the decimal comma are known as decimal places.

Any fraction other than a decimal fraction is called a vulgar fraction.

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS:

You will realise that, when performing ordinary addition and subtraction, it is necessary to arrange the numbers so that all the units are in the same column, all the tens are in the same column and so on. It is equally important when adding and subtracting decimals to be sure to keep the decimal commas under each other. These two Operations of addition and subtraction can then be carried out as with whole numbers.

2xcmgle la
(1) Add together 6,057; 30,0009; 0,00095; 17,003; 485,08005;
0,50036
6,057
30,0009
0,00095
17,003
435,08005
0, 50036
538,64226 ANSWER
(11) Add together 53,715; 147,013; 281; 0,1073; 17 589,9725
53,715
147,013
281
0,1073
17 589,9725
18 071,8078 5g3ggg
Examele 2:
Subtract 34,126 from 675,32
675,320 .... (1)
34,126 .... (2)
641,194 .... (3) ANSWER
Note as a check, by adding mentally lines (2) and (3) line (1) should be obtained.
t;;LTIPLICATION 0F DECIMALS:l
The multiplication of decimals is done in exactly the same way as the multiplication of whole numbers. The number of decimal places in the product is equal to the sum of the number of decimal places in the multiplier and the multiplicand.

Examele 3:
Multiply 53,4 by 2,1
Multiplying 534 by 21 the product obtained is 11 214. There is 1 decimal place in the multiplier and l in the multiplicand and therefore the number of decimal places in the product must be 1 t l or 2, i.e. the result is 112,14. ANSWER..
The two decimal places are marked off from the right.
Note as a check that a rough result would give 53 x 2 : 106 showing that the answer would be somewhere in the region of 100. Thus if an answer of say 1121,4 was obtained it would be obvious from the rough answer that an error had been made. This rough check is of importance in all practical calculations.
The multiplication of a decimal by 10 or a power of 10 is of great importance. When a decimal is multiplied by 10 the decimal comma is moved one place to the right e.g. 0,236 x 10 : 2,36.
When multiplied by 100 the decimal comma is moved 2 places to the right and similarly for other powers of 10. ’
e.go 0,23659 x 100 : 0, 23659 x 102 or 23,659
0,23659 x 10 000 0,23659 x 2104 61- 2365,9
Conversely dividing a decimal by 10 moves the decimal comma cne place to the left and similarly for other powers of 10.
e.g. 23,659 e 10 : 2,3659
23,659 % 1000 2 23,659 e 103 or 0, 023659
When three or more decimals are multiplied together the product of two is first found and then the product of this decimal and the third is found and so on. The number of decimal places in the final product is the sum of the decimal places in the separate decimals which are multiplied together.
e.g. 0,25 x 1,1 x 1,3
By ordinary multiplications 25 x ll x 13 3 575. The number of decimal places in the result will be 2 e l # l : 4. Marking off 4 places from the right the answer is 0,3575.

Rough check 0,25 x l x 1,2 : 0,3

DECIMAL PLACES AND SIGNIFICANCE FIGURES:

It is possible by means of accurate instruments to measure a length to three decimal places. If then an area i.e. the product of a length by a breadth is obtained the result is found to have six decimal places. Because it is not necessary to express an area so accurately the result is given correct to a number of decimal places. The approximation consists of rejecting decimals and the result is given to the number of places required. The rule adopted is as follows:-

If the figure which is rejected is greater than or equal to 5 the preceding figure is increased by one.

Thus 32,64749 is 32,6475 correct to 4 decimal places
32,648 correct to 3 decimal places
32,65 correct to 2 decimal places
32,7 correct to 2 decimal place.

If no figure precedes the decimal comma, significant figures do not include any noughts which occur between the decimal comma and the first figure.

Thus, 0,0000123 is correct to 7 decimal places but only to 3 significant figures, the significant figures being 123.

In this case the result correct to 2 significant figures would be 0,000012.

DIVISION OF DECIMALS:

The important points to remember in dealing with the division of decimals are:-
(0) always make the divisor a whole number
(b) always work out a rough answer.

Examele 4:
Divide 4,321 by 2,47
First make the divisor a whole number. To do this, in this case, multiply both decimals by 100. This gives 4,321 % 2,47 2 432,1 5 247 and proceed to divide in the usual way. Note that a rough answer is 4-;2:2.

247)432,1 (1,749
247
A1851
1729
1220
988
2320
2223 and so on

The answer correct to 2 decimal Blaces is 1,75.
Note how this compares with our rough answer 2. An alternative method is to arrange the divisor as a whole number and place the quotient above the dividend when the decimal point in the quotient will be exactly above the decimal point in the dividend.

Examele 5:
Divide 69,173 by 3,13
Arrange the divisor as a whole number, i.e. 69,173 % 3,13 : 6917,3 e 313
Note a rough answer is 69 e 3 : 23.

22 1
313 6917,3
626
657
626
313
313 The result is 22,1 ANSWER.
__

tCONVERSION OF A VULGAR FRACTION 10 A DECIMAL FRACTIONzl
L_ "_,___.._
denominator in a vulgar
o be divided by the denomina-
decimal fraction is therefore
d give your result correct
5 works out exactly.
The line separating the numerator from the
tlaction indicates that the numerator is t
tor. The rule of converting a vulgar to a
to divide the numerator by the denominator on
to a suitable number of places unless the calculation
Examele 6:
14 .
Express 53 as a dec1mal.
Arrange the division as in Example 5.
O 56
2551 ,00
1 5
_.__
N-b
8;
5
H H
O
ANSWER.
0:56
Note here the inclusion of the 0 before the decimal comna. This shows
that there is no whole number and also fixes definitely the position of
the decimal comma.
From the definition of a decimal fraction the following results may be
directly obtained:-
L 0 _l_ _ . 1
10 0,1 ; 100 _ 0,01 , 1000 : 0,001
Cvriain importa1t vulgar fractions which occur quite frequently in
practice should be carefully noted together with their corresponding
decimals.
0,375
: 0,625
COth
(IJIV
: 0,125 g 1 0,375
CDIH
i _ 5 i _ 3
4 _ 0,25 2 _ 0,5 I : 0,75

REClRRIm DECIMALS:
It sometimes happens that when changing a vulgar fraction into a decimal ’ an exact result is not obtdined, in fact a set of numbers will be found to repeat as long as the division is carried out. Such a decimal is known as a recurring, repeating or circulating decimal.
Examele 7:
Express %3 as a decimal.
0 2424
335 8,00000
h-h10x
$80
mgg’gla 8’
This will give a result %3 : 0:242424 ..u. and so on ANSWER
In order to simplify the setting down of a recurring decimal the following conventional method of expressing this is adopted
g5 : 0,5i, the dots over the 2 and 4 indicating that the 2 end 4 both recur. Similarly i : 0,$ the dot showing that the 3 recurs and i :
, 3 7
0,14285; the dots showing that all the figures between the two dots recur i.e. 142857 recurs.
Recurring decimals in which all the figures recur are known as pure recurring decimals. Mixed recurring decimals are those containing a non-recurring and a recurring part.
l
6.9. $2 : 0,083 is a mixed recurring decimal since only the 3 recurs.

Recurring decimals may be added or subtracted in the usual way, sufficient figures being put down to cover the number of places required.

Examele 8:

Find the sum of 0,1i3, 3,?40 and 2,0 correct to 5 decimal places"

Because a result to 5 places is required, work to 6 (i.e. one more).

0,123232
3,743743
2 333333
__L..._.__._
6 200308
._J__.__.__

The result is 6,20031 (Egirect to 5 decimal Blaze; ANSWER

When multiplying or dividing recurring decimals the simplest method is to change them first into tneir corresponding vulgar fractions and then perform the operations required finally converting back to decimals.

CONVERSION OF DECIMN_S T0 VULGAR FRACTIONS:

1
You have already seen that 0,1 is a method of writing I6 also 0,01 :
l
100 etc. The following rule can therefore be given for changing a a decimal to vulgar fraction: Use the given decimal as the numerator and for the denominator write down a 1 followed by as many noughts as there are decimal places in the given decimal fraction. If necessary then reduce this faction to its lowest terms.

_ _l11 w _l& 3 561
e.g. 0,144 _ - 125 0,3561 ; 10 000
1000
0,0031 : 160%06

This rule applies cnly to ordinary decimals and there are special rules for pure recurring and mixed recurring decimals.

For pure recurring decimals the following rule is used:-

Write down as numerator the figures that recur, and as denominator as many nines as there are recurring figures.

e.g. 0,24 : 2f : g. (See also the converse operation in
99 33
Example 7)
’ 3 l ’ ’ 142857 1
0,3 : 5 : 3 , 0,142857 : 999999 - 7
" 9 l
0,09 : 5; : ll
These last three are important and should be carefully noted.
For mixed recurring decimals the following rule is used.
To obtain the numerator subtract the non-re0urring figures from all the figures, and for the denominator write down as many nines as there are recurring figures followed by as many noughts as there are non-recurring figures.
mgmn6:---:-:- :- mwm
900 900 36 12
" _ 3242 - 32 3210 107
0,3242 - 9900 9900 : 550 ANSWER
916 - 91
To show that 0,916 :
then by 100 and subtract.
0,916 x 1000 : 916,6
0,916 x 100 91:6
.’. 0,916 x 900 : 916,6 - 91,6 : 916 - 91
. . 916 - 91
. . 0,916 : 900
THE METRIC SYSTEM:
This system, now widely used is based on decimals and one of its greatest advantages is that one definite unit is taken for each set of measures and all others are powers of ten of this unit. The prefixes most likely to be met and their meanings are as follows (abbreviations are shown in

deci (d) ; %0 of unit
centi (c) : 100 of unit
. . g 1 .
M1111 (m) _ 1000 of Uhlt
the above are the more common, but tan other prefixes that may be met are
mega (M) : l 000 000 times
Micro (P) : Wof unit
The basic units are metre for linear measure, gramme (or gram) for mass (wcight) and litre for capacity.
The money system uses rands (R) and cents and R1 : 100c.
An exception to the metric system is the measure of time, the relatione ships being:
60 seconds : 1 minute
60 minutes : 1 hour
24 hours : 1 day
7 days : 1 week
365 days : 1 year
Sometimes it is necessary to express one quantity in terms of another quantity of the some kind. This, using the metric system is quite simple, merely requiring shifting the decimal comna; but when dealing with time the tabulated conversion factors must be used.
Example 9:
Express l,573 kilometres (km) as centimetres (cm)
1 km 4 1000 metres (m)
l,573 km 2 1573 m (shifting decimal comma three places to the right)
I m 2 100 cm
1573 m 4 IS? 300 cm ANSWER
gxumgle 10:
The measurement from a scale drawing represents 1 342 000 millimetres (mm).
What is this in metres and kilometres?
2
.5

To convert mm to m we must divide by 1000, i.e.move the decimal
comma 3 places to the left, thus:
l 342 000 mm : 1342,000 or 1342 m ANSWER (L)
Because 1 km : 1000 m
1 342 m : 1,342 km ANSWER (2)
(Decimal comma is again moved 3 places to the left to divide by 1000
when converting metres to kilometres).
Examele 11:
A tank contains 17 hectolitres of wine. Neglecting any losses, how
many litre bottles could be filled?
100
1700 litres
1700 bottles ANSWER
Hecto
’. l7 hectolitres
II II II
Examgle 123
(1) Express 403,20 minutes as a decimal of a week
(ii) How many seconds are contained in this time?
403,20
(1) 403,20 min : 6O :
6,72 h (to convert minutes to hours
divide by 60)
9422 0,28 days (to convert hours to days
24 divide by 24)
: 9#gg : 0,04 week ANSWER
(Divide by 7 to change days to weeks)
(ii) 1 minute 60 seconds
403,20 min 403,20 x 60 25.;92 seconds ANSWER
Examele 13:
Express 0,0375 of a week in minutes

. . 0,0375 of a week :
No. 1.
No.2.
’3
0,0375
7
0,2625
4
1,0500
6
6,3566
60
378,0

days
hours
min.
378 min. ANSWER
-------- oOo--4-----
lPROGRESS QUESTIONS!
Find the sum of:
(a) 0,234 232,15 0,00857 5607,25
(b) 34,005 14,94 0,001304 530,00
Subtract:
(a) 57,704 from 713,00683 (b) 27,148 from 9816
Multiply:
(a) 81,432 by 0,0378 (b) 4,07 by 2,052
Divide:
(0) 112,14 by 0,0534 (b) 0,429408 by 59,64
5
(0) Express 13;
correct.
(b)
Reduce to a decimal
Guns
as a decimal and prove that your result is

No. 6.
E
FRACTIONS. PAGE 13.
Reduce to vulgar fractions:
(0) 0,325 (d) 1,127
(b) 0,0375 4 (e) 0,81
(c) 5,624 (f) 4,7622
Express in minutes:
(a) the sum, and
(b) the difference between 0,342 of a week and 1,63? days.
.... 000------..-
ANSWERS T0 PROCESS QUESTIONS
(a) 0,234 (b) 34,005
232,15 14,94
0,00857 0,001304
5607,25 530,00
5839,64257 ANSWER 578,946304 ANSWER
(a) 713,00683 (b) 9816,000
57,704 27,148
655,30283 ANSWER 9788,852 ANSWER
(a) 81,432
0,0378
651456
570024
244296
3,0781296 ANSWER
81 x 4 324
Rough check 81 x 0,04 _ 100_ : m : 3,24
(b) 2,052
4,07
14364
8208
8,35164 ANSWER Rough check 4 x 2 : 8

E
FRACTIONS. PAGE 14.
(a) 112,1400 5 0,0534 : 1121400 ; 534
2100
53451121400
1992
534
ggg
ANSWER 2199
. 5 100
Rough check 112 7 100 : 112 x 5 ; 112 x 20 _ 2240
(b) 0,429408 f 59,64 : 42,9408 % 5964
0 0072
5964542,9408
41,748
1,1928
1122.9.
. .... ANSWER 0,0072
Rough check 0,42 % 60 : 0,042 l 6 : 0,007
(0) 1% : 0,416 ANSWER
1 416 0 41
Pro of 0,416 : -200...
$152
_ 900 2 33 " 12
7 9
0 __ 1 -0
(h) 2 1 2
5 8 10 32
The L.C.Mu of the denominators is 160
31.1.0.9 11.242 821 1
160 1 160 1 160 1 160 : 22_’
136 160

FRACTIONS. PAGE 15.
0 00625
160$1,00000
960
400
w
800
800
2,00625 ANSWER
325 13
No. 6. (a) 0,325 : 1650 : Z2
(b) 0,0875 : 875 _ gg_ _ z_
10000 _ 400 7 g_
524 156 78
(C) 5,624 : 5m : 5 % : 51-72
" 127 - 1 126 14 7
(d) 1,127 :l 79?)- : 1-99_O : 1m : IE
" 81 9
(8-) 0,81 :: E : E
’ ’ 7622 - 7 7615 1523
(f) 4,7622:4_’5560_ : 4W :41???
No. 7. 0,342 week 1,637 days
7 4
2,394 days 6,548
4 ___Q
9,576 39,288 hours
___;2 60
57,456 hours 2357,280 min.
60
3447,360 min.
Sum : 3447,36 4 2357,28 : 5804,64 min. ANSWER (a)
Difference : 3447,36 : 2357,23 : 1090,03 min. ANSWER (b)
50 811

ayERAGES; RATIO AND PROPORTIGQ; PERCEleGES

INTRODUCTION:

In this lesson, we shall deal with Averages, Ratio and Proportion, and Percentages, and it will be seen haw the basic rules and operations are applied.

While these topics are not difficult in themselves, you are advised to study the lesson very carefully and to make sure that its subject matter is clearly understood. The methods considered here will be used constantly in later work.

AVERAGE:

The average of any series of numbers or quantities is the mean value of those numbers or quantities. The rule to be adopted is ’Divide the sum of the numbers or quantities by the number of numbers or quantities’.

Examale 1:

Find the average of the numbers 0,36; 0,48; 0,16; 0,32; 0,76.
The total here is 2,08 and dividing by the number or numbers, i.e. 5, the average value is 0,416. ANSWER

Examele 2:

The mass of eight men in a boat’s creu are 60,00; 62,10; 64,20; 68,15; 65,00; 70,01; 63,76 and 64,46 kilograms. What is the average mass?
Adding the mass together the sum is 517,68 kg.
The average mass therefore is 517,68 4 8 : 64 71 kg; ANSWER.
number of men.

special case is that in which the numbers under consideration, when placed in order of magnitude, increase uniformly.
Such numbers are said to form a series. The average of the whole series is the average of the first and last terms in it.

ExcmEle 3:
Find the average of 5, 8, ll, l4, 17, 20.
In this case each number differs from those on either side of it by 3.
The average is 5-3-33 : 12y2 ANSWER
Nate that using the method previously outlined the average is
54-8-6-11-1-14 e17 e20: 22 : 12/2
6 6

Errors are often made by adopting the following method of finding the average of a series of numbers. Divid;ng the series up into a number of groups, finding the average of each group and then finding the average of these averagesu The answer thus obtained is incorrect when it applies to speeds. The only correct method of finding an average speed is to divide the TOTAL distance by the TOTAL time irrespective of any variation of speed.
Note how this is demonstrated in the example which follows:-

ExcmuLe 4:
.J.
etaimine the average speed of a train moving 105 metres in 15 seconds, 90 metres in 9 seconds and 38 metres in 8 seconds.
Total distance : 105 e 90 t 88 : 283 metres
Total time : 15 e 9 e 8 : 32 seconds
283
Average speed : -53 : 8:84 metres Eer second ANSWER ... (a)
Note that the incorrect method would be

________J___________.___________L______________.___.____
105 metres in 15 seconds
90 metres in 9 seconds
88 metres in 8 seconds
.'. Distance in 3 seconds
Average speed : gg :
7 metres in 1 second
10 metres in 1 second
11 metres in 1 second
7 t 10 e 11 : 28 metres
9:3 metres Ber second .... (b)
To check on average remember that the average multiplied by the number of terms gives the total. Using result (a) the total distance is given by average multiplied by time : 8,84 x 32 : 283 metres, which as has already been seen is the total distance. Using result (b) total distance : 9,3 x 32 : 299 metres approximately. Note that this latter method gives an incorrect result, and that this method must not be used.
RATIO:!
It is often necessary to compare two numbers or like quantities. Consider the two numbers 12 and 8. These may either be compared by stating their difference or by expressing the number of times one is contained in the other. In this case the difference is 4 and 8 is contained in 12, 1V2 times. Now consider the two numbers 12 and 16; their difference is 4 but 16 is contained in 12 only 3/4 times. It is obvious therefore that the difference may lead to some ambiguity and the other method is used. This wis known as a ratio and the ratio of one quantity to a second is the number of times the first quantity contains the second. In other words a ratio is a fraction with the first quantity as the numerator and the second as the denominator.
e.g. the ratio of 3 to 2 is written as:
%0r3%20r3:2
The colon between the two numbers indicating that the ratio is required.
The conception of a ratio can only be used when comparing two quantities 3f the same kind; as, for example, two lengths, two weights, two Ielocities, two coins etc. It is_meoningless to speak of the ratio of one elephant to one apple; on the other hand, the ratio of the mass of an elephant to the mass of an apple or the volume of the former to that of the latter, has a definite meaning.

5
Thus the ratio of 5 kilograms to 8 grams is not 5 z 8 or 5 but
I
5 x 1000:: 8, that is, 5000 : 8 or 625 : 1.
When an answer which is a ratio does not contain simple numbers only,
it is usually convenient to express it as a ratio in which one term is 1.
Thus if an answer 2,7 : 3,8 was obtained this would be written 1 : 1,4 0:
0,71 : 1.
To divide a given number in a required ratio add the two terms of the
ratio to obtain a common denominator and take each of the terms in turn
as numerators.
Examgle 5:
Divide R70 in the ratio of 2 z 3
Here the denominator becomes 2 e 3 : 5
g of 70 z 28 and g of 70 z 42
The required amounts are therefore Egg and Big ANSWER
When dividing a given number in the ratio of two fractions these must
first be reduced to equivalent fractions with the some denominator.
Examele 6:
. . . . l l
D1v1de R90 1n IOth 2 . 3
. l l . . l l
Th1s does not mean I of R90 and 3 of R90. Wr1t1ng I and 3
as fractions with the same denominator R90 must now be divided in the
at. i c L
r 10 20 . 20 .
i.e. R90 must be dilided in ratio 5 : 4.
Here the denominator becomes 5 e 4 : 9
g of R90 : R50 ahd % of R90 : R40

___._____J_______._________________L________________.__n

The required results are Egg and Big: ANSWER.

Alternatively using the general method the following solution is obtained:-

Divide R90 in the ratio of % ; %
1 9

Here the denominator becomes % e 3 : 26
1

gofR90:%xg-ox90:50
50
1

g of R90 : % x 2g x 90 : 4O
25 m

Hence the required amounts are 529 and Big AE3ggE.

PROPORTION:

Proportion indicates the equality of two or more ratios. It is evident that 2 : 3 and 12 : 13 are the same. The equality of the two ratios is expressed as a preportion in the following manner 2: 3 : 12 or 2 : 3 :: l2 : 18, the double colon or the equals sign indicates that the two ratios are equal. The four terms are said to be proportional and the proportion is read as '2 is to 3 as 12 is to 18'.

The first and last terms of a proportional are called the extremes while the second and third terms are called the means. In this case the 2 and 18 are the extremes and the 3 and 12 are the means. The relationship between the extremes and the means is most important and may be expressed as follows:-

The product of the extremes is equal to the product of the means. In this case 2 x 18 : 3 x 12. It follows that if-three terms of a proportional are given the fourth can be found.

Exoggigai:
Find the first term of a proportional in which 5, 6 and 15 are the remaining terms.
i.c. lst term : 5 : 6 z 15
lst term x 15 : 6 x 5
5 x 6
lst term :2 _...__2 : 2
15
1.9. the complete proportional is 2 : 5 : z 6 : 15 ANSWER
leAN PROPORTIM:
When the second and third terms are equal they are said to be the mean proportional of the extremes.
0.9. 2 : 8 : 8 : 32
Here the 8 is said to be a mean proportional to 2 and 32.
Note here the important point that the mean proportional to two numbers is found by faking the square root of their product.
i.o. in this case the mean proportional to 2 and 32 is:
J2 x 32 :6/64' : 8
Hum BROPORTIONAL-
The third proportional to two numbers is the number which bears the same ratio to the second as the second boats to the first.
e.g. the third proportional to 2 and 6 is 18
because 2 z 6 : 6 : 18

FOURTH PROPORTIONAL:
Similarly the fourth proportional of three numbers is the number which bears the same ratio to the last of the three as the second bears to the first.
e.g. the fourth proportional to l, 2 and 6 is 12 because
1 : 2 : 6 : 12.
DIRECT AND INVERSE PROPORTION:
One quantity is said to be directly proportional to or ’vary directly’ as another, when an increase in value of one produces an increase in value of the other. For example, if one book costs R3, then two books cost R6, and three books cost R9 and so on. In other words the cost varies directly or is directly proportional to the number of books.
Alternatively one quantity is said to be inversely proportional to, or to ’vary inversely’ as another when an increase in the value of one produces a decrease in the value of the other. For example, if one man can do a piece of work in a certain time, then two men could do the same amount of work in half the time, and three men in one third of the time and so on. In other words, the time taken is inversely proportionsl to the number of men working.
USE OF PROPORTION:
Very many of the problems which occur every day can be solved by the use of proportions. IF three of the terms are given the fourth can be found.
Examele 8:
If 60 men do a piece of work in 40 hours how long will it take 100 men to do a similar piece of work?
h
the ratio of $93
60
Here because the number of men has been increased in
it is obvious that less time be required.
As the time is inversely proportional
to the number of men employed the
time taken:
: 40 x $66 : 24 hours ANSWER.

Examgle 9:
If 25 metres of cloth cost R37,50 what will be the cost of 35 metres?
Here the cost is directly proportional to the number of metres which
has been increased in the ratio g2
25
35 75 35
Cost of 35 metres : R37y2 x 53 R_E x 53
: R52z50 ANSWER.
This type of problem can also be solved by the use of what is known as
the Unitary Method.
UNITARY METHOD:
This involves finding the value of one unit and then by multiplication
the value of the required number of units can be found. This method can
be used to solve examples 8 and 9.
gxamgle 8: (Alternative method)
If 60 men do a piece of work in 40 hours
Then 1 man will do the work in 40 x 60 hours .... (a)
3.100 men will do the work in W .... (b)
24 hours ANSWER.
Note at (o) the time is multiplied by the number of men because it is
obvious that 1 man will take longer than 60 men.
And at (b) the time one man takes is divided by 100 because 100 men will
take less time than one man.
Examele 9: (Alternative method)
If 25 metres of cloth cost R37,5O
R37z50
Then 1 metre of cloth will cost 25 .... (a)
.'. 35 metres of cloth will cost Rgzigg X 35 .........-..------..(b)
: R52:50 ANSWER.

_________i___.____________________J..__________.u______u.ii_.

Note at (a) it is necessary to divide by 25 because 1 metre will cost less than 25 metres.

And at (b) it is necessary to multiply by 35 because 35 metres will cost more than 1 metre.

An important point in the majority of examples is to leave all the calculations until the end of the question. It is then possible by cancelling to simplify the calculation which can then be carried out in the normal way.

Examples in proportion frequently occur in which three or more quantities vary together. These may be solved in a similar manner to that outlined above.

ARITtNETICAL AN) GEMETRICAL MEANS:

The arithmetic mean (A.M.) of two numbers is the average of the numbers:

e.g. the arithmetic mean of 6 and 12 is 6 Z 12 : 9c

the arithmetic mean of 4 and 7 is 4 Z 7 : 5,5

The geometic mean (G.M.) of two numbers is the mean proportional of the two numbers

e.g. the geometrical mean of 4 and 9 is. 4 x 9 : 6

the geometrical mean of 2 and 32 is V2 x 32 : 8

PERCENTAGES

In the addition, subtraction and comparison of fractions it was first necessary to change these to new fractions having the same denominator or in other words to express the fractions in the form of similar fractions having a common denominator. A percentage is such a fraction in which this denominator is 100.

Any given fraction may be expressed in the form of a percentage:

thus:

_________L____.________________._L__________._____________
rr
% - fgo or 25 per cent
% ; ?ga or 50 per cent
1
% 3363 or 33V3 per cent

Instead of writing the words 'per cent', the symbol % is generally used, thus 33y3 per cent could be written 33y3%. The following examples show practical instances of the use of percentages. The important point to remember is that a percentage increase or decrease is always taken with references to the original amount. Thus, unless otherwise stated, percentage profit is always calculated on the cost price.

e.g. the percentage profit on an article bought for 10c and sold for 12c is
16 x 100 or 20% and 921 %3 x 100 or 162/3 %

Exomgle 10:
The population of a town was 459 230 in 1953 and 483 450 in 1973. What is the increase and percentage in the population?
Increase : 483 450 - 459 230 : 24 220 ANSWER.
% Increase t Increase
Original population
4 c 3
giazggo x 100 : 5(27% ANSWER.
x 100

A town had 0 population in 1953 of 763 420 and in 1973 of 811 860.
Find the increase and percentage increase in population.
Increase : 811 860 - 763 420 : 48 440 ANSWER.
__ _ 48 440
% Incleosc - 763 420 x 100 : 6:33% ANSWER.

Note here that although the increase in the second case is twice the increase in the first, the percentage increase is not twice as much

Examele 12:
An agent engaged in selling motor cars, receives a commission at the rate of 12%%. What will he receive on a car which is sold for R450?
12%% commission means that he will receive R12%w on every R100
- - _ 123: _ a
. Amount received on R450 _ R450 x 100 _ R56EL5 ANSWERO
The efficiency of a machine is also given as a percentage. In this case the work given by the machine is expressed as a percentage CF the work put into the machine. For example, if two machines have efficiencies of 70% and 80% respectively the second is obviously the more efficient.
In an internal combustion engine, for example, about 33% of the heat available in the petrol when it explodes is turned into useful work, This engine therefore has an efficiency of 35%. It now this drives a machine which is itself 73% efficient then the efficiency of the combination is 75% of 33% : % of 35% : zaggu
It can thus be seen that of the total energy available uhly 26%% does useful work, the remaining 73%% being used t0 overcome resistance and disappearing as heat losses.
Examele 13:
A man sells a piece of property for R504 and makes a profit of 12%. What was the cost of the property?
If the profit was 12% then the selling price was 112% of the outlay.
112% of the outlay : R504
504
1% of the outlay : RITE
.’.1OQ% i.e. the total outlay : R%%% x 100
R450 ANSWER.
Examgle 14:
In what proportion must a merchant mix tea at 60c/kilogram with tea at 38c/kilogram, so that by selling the mixture at 56c/kilogram he makes a profit of 40%? ’

_______L______m_$_n________L__n_______e-_--_---

Because profit is 40% then the selling price is 140% of the cost price.
140% of the cost price : 56c
. 56
1% of cost price _ 140
100% of cost rice : 7g- x 100
P 140
: 40c.
Loss on each kilogram of tea at 60c sold for 40c : 20c
Gain on each kilogram of tea at 38c sold for 40c : 2c
.'.He must mix the teas in the proportion.
T90 at 60c er kilo ram _ 2g _ 1_
Tea at 38c per kilogram _ 20 _ 10 ANSWER-
Note here that the loss on the 60c tea must equal the gain on the 38: tea.

This last example shows the use of both proportion and percentage and numerous other practical examples of these could be given. Because the same principles are involved in all examples on percentage the student should be able to solve these quite easily whenever he comes across them. Bear in mind the very important point that the percentages are always calculated on the original values.

iPROGRESS QUESTIONS.

No.1. An indicator diagram is divided into 10 equal parts. If the pressures in kilonewtons per square metre are 140; 140; 102,35; 77,75; 59,6; 47; 46,55; 38,05; 33,6 and 30, find the mean or average pressure.

No.2. The average speed of a train for 3 h 35 min is 42 km per hour am it is 36% km per hour for the next 1h 48min. Find the total distance travelled.

A does a piece of work in 8 days, B in 12 days and C in 6 days. If a day's work of D is the average of the other three, how long will D take to do the work?
Find the mean proportiohal of 0,9 and 2,5.
A bankrupt owes three creditors R250, R350 and R400, respectively. If R575 is divided among them in the ratio of their claims, how much will each receive and how much will be paid in the Rand?
Divide R202,l6 among three persons in the ratio
COIN
.2
'4
01H?-
A cottage is sold for R16 500 thereby gaining 10%. What did it cost originally? At what annual rental must it be let in order to pay 7% on the purchase money?
(a) What is 5% of R2,90?
(b) _Find the number 33y3 % of which is 40% of 75.
ANSWERS T0 PROGRESS QUESTIONS
Question 1:
An indicator diagram is divided into 10 equal parts. If the pressures in kilonewtons per square metre are 140; 140; 102,35; 77,75; 59,6; 47; 46,55; 38,05; 33,6 and 30, find the mean or average pressure.
Average pressure
1
714,9 _
10
40 e 140 t 102 35 e 77 75 t 59 6 e 47 t 46 55 e 38 05 t 33 6 t 30
_____________J_______L______J__________J.______L_______L______
lO
_ 71,49 kilonewtons/sg.m. ANSWERQ

Question 2:
The average speed of a train for 3h 35min is 42km per hour and it is 36%km per hour for the next 1h 48min. Find the total distance travelled.
Distance travelled : average speed x time
Distance travelled in first period : 42 x 3%2 : 150,5 km
Distance travelled in second period : 36% x 1% : 65,7 km.
.’. Total distance 216l2km ANSWER.
Question 3:
A does the work in 8 days and therefore does % of the work in 1 day
lk-
B does of the work in 1 day
2
and C does
(3le)-
of the work in 1 day
’. A t D t C together do A t i t % : 3 l 2 l 4
a 12 24
: g4 : g of the work in 1 day
D does the average of A,B and C in 1 day
i.e. D does % Q 3 : i of the work in 1 day.
i.e. D will do the work in 8 daxs ANSWER.
Question 4:
Find the mean proportional of 0,9 and 2,5
Mean proportional of 0,9 and 2,5 : y/0,9 x 2,5
2,25 : 1L2 ANSWER.
Question 5:
A bankrupt owes three creditors R250, R350 and R400, respectively. If R575 is divided among them in the ratio of their claims, how much will each receive and how much will be paid in the rand?

R575 is to be divided in the ratio 250 : 350 : 400
The denominator is 250 e 350-t 400 1000
. . . 250
. . 13? person receives I000 x 575 : Rl43,75
2nd person receives 2380 x 575 : R201,25 ANSWER.
3 d c ’ve Q9- x 575 - R230 00
r person re e1 5 1000 _ ,
Note as a check the sum Rl43,75 t R201,25 t R230,00 : R575
. . R575 _ _ 1
Amount paid in R1 E606 _ R0,575 _ 572c ANSWER.
Question 6:
. . . . 2 3 4
D1v1de R202,l6 among three persons in the ratio 5 : Z : 3 .
R202,l6 is to be divided in the ratio % : % : g .
The denominator is z e 2 t i : 59 e ii t 5g - 122
3 4 5 60 60 60 _ 60
37: 4o
’. lst person receives 3 x R202,l6 : - x R202,l6
m 133
736 : R60l80 ANSWER.
-3- 45
’. 2nd person receives 4 x R202,16 : -- x R202,l6
$55 133
60 : R68z40 ANSWER.
1 48
’. 3rd person receives 5 x R202,l6 : --x R202,16
I33 133
736 : R72z96 ANSWER.
Note as a check that the total does equal R202,l6.
Question 7:
A cottage is sold for R16 500 thereby gaining 10%. What did it cost originally? At what annual rental must it be let in order to pay 7% on the purchase money?

_________L_______________________J__________________________
Because there is a gain of 10% then the selling price is 110% of the cost price
i.e. 110% of the cost price
1% of the cost price
.’.100% of the cost price
Annual rental
Question 8:
ll
R16 500
R16 500
110
R16 500
110 ANSWER.
x 100 : R15 000
7% of purchase money
7
R___
100
7% of R16 500 x 16 500
R1155 ANSWER.
(a) What is 5% of R2,90.
(b) Find the number 33y3 % of which is 40% of 75.
5
(o) 5% of R2,9O _ E6 x R2,90
1
: 26 x R2,9O
: 14%c. ANSWER (a).
(b) 40%of75 : 111%)(75 : 30
Now 33V3 % of a number : 3O
. 100
. the number _ 30 x 5573
: 9_o ANSWER (b)
50 813

INTRODUCTION TO ALGEBRA.

A. USE OF LETTERS IN ALGEBRA:

Mathematics is an important science and it is nearly all founded on Algebra. Many problems can be solved by Algebra which it would be impossible to solve by Arithmetic.

Algebra is an extension of Arithmetic. In it we extend the processes and operations of Arithmetic and make them more general. In Arithmetic we use definite numbers and find definite numerical results. In Algebra we mgy use definite numbers but on the whole we deal with general expressions and general results in which letters represent any number.

This may seem vague to a beginner, but if we examine some easy examples in Arithmetic and generalise the results by using letters it may clarify what is meant.

Examgles:

1. In Arithmetic the fact that 2 is added to 7 is represented by 2 t 7, so if x represents any number, then x t 7 is the result of adding x and 7. More generally, if a and b are any two numbers then the addition of a to b is a t b.

2. In Arithmetic, numbers can be added in any order, e.g. 2 t 7 : 7 t 2. In Algebra a t b : b t a, and this form of the statement includes any values you wish a and b to represent.

3. We know in Arithmetic l x 0 : O, 2 x 0 : 0, etc. This in Algebra is stated a x O : 0 where 0 represents 92y number.

4. If a length 4m, is to be cut from a plank 7m long, the remaining piece is (7 - 4)m, i.e. 3m. 50 if the plank is ym, long and x m, are cut off, the piece left is (y - x)m.

5. We know 1 metre : 100 cm. A general statement of this (to change a number of metres to centimetres we multiply by 100) is

xm. x 100 : cm.

6. To find the area of a floor 10m. long and 9 m. broad we multiply. 50 area : 10 x 9 sq.m. This statement using letters sould be A : l x b. The letters 1 and b represent any numbers providing they are in the some units.

7. The distance travelled by a train in 3 hours at 30 k.p.h.
is 3 x 30km.
Algebrically we can say
Distance : t x s
where t is the time taken to travel that distance at a
speed of s.
8. We know that 2 x l e l is an odd number, 2 x 2 s l is an odd
number, 2 x 3 e l is an odd number, etc. All these facts can
be stated simply in the general statement that 2n t l is an
odd number if n is any whole number.
9 In Arithmetic we learned a rule for cancelling fractions, e
2 x 3 2 7 x 5 7
: - 3 x 7 3 : -
5 3 5 ’ : , ’
x 4 x 7 I 5 x 8 8
The general statement of this rule is
a x y _ g
b x y ’ b
whereby a, b and y represent any numbers.
. . y a t y _ 3
Similarly, b % y _ b
QUESTIONS FOR PRACTICE: (Answers at end of lecture).
1. Write down: (a) Number of metres in n kilometres.
(b) Number of metres in x centimetres.
(c) Number of months in y years.
(d) Number of cents in a Rand.
2. If you run 7m. per sec. how for would you go in (a) 10 secs.
(b) t secs?
3. IF eggs cost 5 cents each how much will (a) 12 cost, (b) n
cost, in cents, then in Reno?

4. In cooking meals for a large camp %kg. potatoes is allowed
per person. How many kg. are needed for (a) 40 people,
(b) x people?
5. A boy is 18 months old. How old will he be in (a) b years,
(b) 2 months (answer in years)?
6. (a) A man walks x hrs. at y k.p.h. How far does he go?
(b) A man walks 0 minutes at b k.p.h. How far does he go?
7. At a football match 9 people each pay h cents for admission.
Give the total receipts in (0) cents, (b) Rand.
Write down general statements for the following:-
8- lAl:2xl 9. 5Al:lA5 10. R5:5x100cents
2 A 2 : 2 x 2 3 A 7 : 7 A 3 R3 : 3 x 100 cents
3 A 3 : 2 x 3 11 A 2 : 2 A 11 R17 : l7 x100 cents
9 A 9 : 2 x 9 37 A 9 : 9 A 37
3 2 10 5’
ll. 2 x 3 _ l 12. 10c _ I66 R 13. 5% of 27 :166 x 27
i Z- -3L ’3
7 x 4 _ l 30c _ 100 R 3% of 19 _ 00 x 19
E 2- _Q_ 1 2%
9 x 5 _ l 29c - 100 R 25% of 20 :166 x 20
14. The sum of the first two odd numbers is 2 x 2.
The sum of the first three odd numbers is 3 x 3.
The sum of the first four odd numbers is 4 x 4 .
15. iof5:1 l6. lAlAl:3xl 17. 48mths:-A&
5 12yrs
lcf7-1 24,2ar2-3 2 5th -5-
7 _ _ x m s _ 12 yrs
i _ _ -3
8 of 8 _ l 3 A 3 A 3 - 3 x 3 78mths _ 12yrs

18.2x%:l l9.lxl::12 20. 1x1x1_13
3x%:l 2x2:22 2x2x2z23
4x%:l 3x3:32 3x3x3:33
4x4:42 4x4x4:43
21. V2 x 2 : 2 22. 3&3 : 1 23. 2 x 1 is an even number
J3 x 3 : 3 353 _ 2 x 2 is an even number
V4 x 4 : 4 353 _ 3 2 x 3 is an even number
/5x5:5 wyt
24. 3 times 1 4 l 4 times 1
3 times 5 4 5 : 4 times 5
3 times 19 4 l9 : 4 times 19
B. NOTATION:
The symbols 4, -, x, e, have the same meaning in Algebra as in Arithmetic, but it is not always necessary to write the x sign. The product of 3 and 5 in Arithmetic is written 3 x 5. The product of a and b in algebra is written a x b, a. b or simply ab. Thus, if 0 stands for 3 and b for 5, ab means 3 x 5, 22: 35.
Similarly, the product of a number and a letter, 3 x a, may be written 30; always place the number before the letter, thus 30 not 03. Since multiplication by 1 leaves the number unchanged it is unnecessary to write the l in such a product, so the expression la is never used, but merely a is written.
The symbols for roots and powers also have the same meaning in Algebra as in Arithmetic. 34 means 3 x 3 x 3 x 3, and a4 means a x a x a x a. 3V/3 means the cube root of 8 and 33/; means the cube root of a.

If a number can be expressed as the product of two equal factors, one of these factors is said to be a square root of the given expression, thusx/gz : 8. If the expression is the product of 7 equal factors, one of these factors is called a seventh root of the given expression, 71/128 : 2 1 If the expression is the product of n equal factors, one of those factors is the nth root of the expression nVc:n : a.
The symbols (and) mean 'less than' and 'greater than', e.g. x(2 means x is less than 2, x)2 means x is greater than 2, x42 means x is equal to or less than 2, and 3()((5 means x is greater than 3 but less than 5.
Examele 10:
What is the meaning of 2x - 1? What is its value when x : 3?
This expression means that a number x is multiplied by 2, and l is sugtracted from the product. When x : 3, 2x - l : 2 x 3 - l : 6 - 1
(This value could, of course, have been worked mentally).
Examgle 11:
If a z 1 and b : 2 what is the value of 2a2 4 Gab?
2a2 t 3ab : 2 x 12 t 3 x 1 x 2 : 2 1 6 : 8,
(N.B. l: : l and all powers of l are 1. This may be generalised as 1 :1).
QUESTIONS FOR PRACTICE:
25. If a : l, b : 2, x : 3, y : 0, find the value of the following:-
(i) a t b, (ii) b 1 0 (iii) ab (iv) xy
(v) 3a2 (vi) 2x3 (vii) 2&5 (viii)
X-Cl
b
N
0
H
'1'!
X
ll
2, find the value of the folloWing:-
(i) x2 _ 1 (ii) 3x2 _ 2x (iii) x3 1 3x t 1
(iv) x212 (v) 1% (vi) %
X

(vii) x5 (viii) 5x
27. If x : l and y : 9, find the value of:-
(1) xy (ii) x3y (iii) 3xy2 (iv) 3 VC5"
(v) 3 v5? (vi) v12? (vii) 25_%_x (viii) % - x
Study carefully the working of the following examples:-
Examele 12:
a 3 b is the square root of 02 3 20b 3 b2. Show that this is true
when a : 5 and b : 3.
02 3 20b 3 b2 : 25 3 2 x 15 i 9 : 64
a 3 b : 5 3 3 8, and 82 : 64
Examele 13:
2 - y2. Show that this is true when x : 7,
"-0-
x y is a factor of x
y 4.
x2 - y2 : 49 - l6 : 33
x 3 y : 7 3 4 : 11. Factors of 33 are 3 and 11.
Examele 1 :
If the diameter of a circle is d cm. its area is approximately
%%d2 square cm. Find the area of a circle whose diameter is
5cm. to one decimal place.
Area : %% x 25 sq. cm2
: % sq. Cm2
: 19,6 sq. cm2 to one decimal place.
(N.B. The unit of area must not be forgotten).

C. ADDITION AhD SUBTRACTION:
1. Like Terms:
Consider the following:-
3334-3333333z3x6
74-7i-74v74-7-l-7:7x6
In the same way
x 3 x 3 x 3 x 2 x 3 x : x x 6 or 6x and
y 3 y 3 y 3 y 3 y : 5y.
QUESTIONS FOR PRACTICE:
28. Write down the value of:-
(i) a 3 a.y a 3 0, (ii) b2 3 b2 3 b2 3 b2 3 b2;
(iii) ab 3 ob, (iv) x y x 3 x 3 x y ...to 17 term
We can continue this process for any number of terms and may extend it to include subtraction, e.g.
3 3 3 y 3 3 3 y 3 - 3 - 3 5 x 3 - 2 x 3 : 3 x 3
H
7 3 7 3 7 3 7 3 7 3 7 - 7 - 7 - 7 : 6 x 7 - 3 x 7 : 3 x 7
x 3 x 3 x 2 x 3 x - x - x : 5x - 2x : 3x
29. Write down the value of the following:-
(i) 4 x 11 y 8 x 11 (do not work out)
(ii) 4x .u 8x
(iii) 7x 3 5x
(iv) 7 x 13 - 2 x 13

(v) 7 x o - 2 x 0
(vi) 1102 - 302
(vii) 13 x 2 - 2
(viii) 13x - x
(ix) xy o l3xy
(x) lZob - 7ob

These examples all ha e a simpler form because the terms are alike, i.e., x t x, a and 02, ab and ab. In this connection you must remember that be is the some as ab (being the product of a and b), and, therefore, ab and be are 'like' terms.

Only like terms can be added and subtracted in this way to give a simpler form. This process is known as collecting teIMs.

Unlike Terms.

Terms which contain different letters or different powers of the same letter are called unlike terms and these cannot be collected to give a simpler form. If you add x to x, the result is 2x; if you add x to y the result is x t y whigh has no simpler form. Similarly the result of adding x and x is x t x2 and of adding x and 2 is x t 2.

It is imperative that you grasp this now, and you must be careful not to confuse addition with multiplication.

x t y has no simpler form but x x y is written xy.
2 has no simpler form but x x x2 is x3.
xtx
x # 2 has no simpler form but x x 2 is 2x.

The same rule of like terms applies to subtraction
5x - 2x : 3x but 5x - 2y has no simpler form.
5x2 - 2x2 : 3x2 but 5x2 - 2x has no simpler form.
2
x - 2 also has no simpler form, nor has x2 - x.

You can only collect terms if the letters are the some and the Bowers are the same. xy and yx are like terms and can be collected, x
t yx : 2xy (or 2yx). 3xy - yx : 2xy
(or 2yx); but x y and y2x are not alike and cannot be collected.

QUESTIONS FOR PRACTICE:

Now work the following, writing down the answers only and putting 'no simpler form' or simply a dash if you think the terms cannot be collected. Work them all before looking for the answers at the end of this lecture.

30. Simplify:-
(i)
(iii)
(V)
3x e 2x e x
5x - 3x e x
7xy - 2x e y
31. Simplify:-
(i)
(iii)
POWERS:
3a - 20
x e y e xy
x e l e x e 2 e x e 3
x e 2y - x 2
30 e 2b e u e b
3x t 2y e y
5x - 3x - 2y
7x2-2x2-i-5x-5
a e a e a e b - 3
x e x e x e y e y ey
a e 2b e 3o e 4b e 5
30b - 3bc - bu
x2 - 2y2 e 322 - x2
e 3y2 - 22

If an answer to a sum contains several different letters, the terms should be arranged in alphabetical order; this is not essential, however, and if the term in a is negative it may be left till later, e.g., -30 t 2b can be written as 2b - 30; but you should keep some order in your answer, thus:
20 - 2b e c - d e 3x - y is better than 20 e c e 3x - 2b - d - y.

If the answer contains several different powers these should be arranged either in ascending or in descending order. Remember that the numerical term is the 0 power and that x is really xl. Thus the correct order of 30 t 02 - 203 e 404 - 1 is:
404 - 2G3 t a2 t 30 - l in descending order
4
or -1 t 30 t 02 - 203 t 40 in ascending order
and the expression has no shorter form.
Notice that each term has its own sign e-or - and that these naturally cannot be transferred: the numerical term is - 1, whether it comes first or last. This numerical term is sometimes called the constant term since its value is quite independent of the value of x.
Study carefully the following example:
Examele 15:
Arrange the following expressions (i) in ascending order, (ii) in descending order, and find their value when x : 10.
(i) 3x2 e 5x t x3 t 2 (ii) 1 t 3x2 t 2x t 5x4 e 4x3
(iii) 5 - x2 e 2x4 - 3x e x3 (iv) x5 t x3 e x - l - x2 - x4
The answers are as follows:-
(i) 2 t 5x t 3x2 t x3 : x3 t 3x2 t 5x t 2 : 1 352
(ii) 1 t 2x t 3x2 t 4x3 e 5x4 : 5x4 t 4x3 t 3x2 t 2x t l : 54 321
(iii) 5 - 3x _ x2 e x3 t 2x4 : 2x4 t x3 - x2 - 3x e 5 : 20 875
(iv) -1 t x - x2 t x3 - x4 t x5 : x5 - x4 t x3 - x2 t x - l : 90 909

_________________._______________

QUESTIONS FOR PRACTICE:

32. Arrange the following in descending powers of x and find their value when x : 10:-

(i) 4x3-i-3x24-5x4-r2x4.6x54-1
(ii) 2x2   3x3   x   4x4
(iii) 2x3   4x5 - x2 - 3x4
(iv) x4   x2 - l   x   x3

33. Simplify the following where possible:-

(i) a34.024-a-I-024-11-a
(ii) b4   4b   b2   2
(iii) c2   0
(iv) 2d _ d2
(v) 2x2 - 2   3x - 3
(vi) y3   2y3   3y3
(vii) 4x2   3x   2 - 2x   3x2
(viii) x2 - 1   x   2
(ix) x2   2x   xy
(x) x3 - 3xy   2y

Do MULTIPLICATION AhD DIVISION:

1. MULTIPLICATION:

x x y x z : xyz, x x 2 : 2x and x x x2 : x3.
or %x, x % x2 :
NIX
. . , x
In the same way x 7 y : 5, x T 2 : 2
y x
l
and the factor x cancels leaving ;.
Remember any number goes into itself once. Do not imagine that 6 e 6 : O; 6 x l : 6, therefore 6 f 6 : 1.
There is only one number which has a factor 0 and that is 0 itself. This cannot be sufficiently emphasised. There are those who believe that if you multiply 0 by a large enough number you will get something, but even a million times 0 is still 0.
The multiplying of different letters is easy: you simply write them together with no sign, a x b x c x d : obcd. The numerical factors are produet, e.g., 30 x 2b x 4c x d : 24 obcd, 3xy x 2 : 6xy. (The word ’literal’ means ’expressed by letters of the alphabet’; e.g., in the examples just quoted abcd and xy ore literal products).
The rule for multiplying together powers of the same letter is add the indices. The reason for this is obvious, since the index gives the number of factors: x4 means 4 factors of x, x3 means 3 factors of x. Therefore, in x4 x x3 there will be 4 factor3 of ; plus 3 factors of x or 7 factors in all, hence x4 x x : x .
Similarly, x5 x x2 : x x x x x x x x x x x x x : x7
5x5 x 2x2 : 5 x x x x x x x x x x x 2 x x x x
: 10x7
Be sure to multiply together the numerical factors, e.g., 30b x 202b is 3 x a x b x 2 x o x a x b : 603b2. Do not actually write out the factors in full in this way. Imagine them written down and merely write the answer.
Examgle 16:
Study the following carefully and make a list of those sums which are worked wrongly. The corrections are given below, but work them for yourself first.

(a) 3x3 x 4x4 : 7x7 (b) 2x5 x 5x2 : 10 x10
(c) a x 20 : 302 (d) a3 x 03 : a6
(e) 3e x a2 z 303. (f) 20 x 02 : 303
(9) 30b x 20b2 2 602b3 (h) 20 x ab : 202b
(i) x x x2 x x3 : x6 (j) 2x x 3x x x : 6x3
Numbers (0), (b), (c) and (f) are wrong, the correct
answers to these are 12x7, 10x7, 202, 203.
DIVISION:
Since division is the opposite of multiplication, the rule for
division is to subtract the indices, e.g., x7 t x3 : x2, y9 t
y4 : y5. You can easily verify this by writing the expression
in full and cancelling:-
x7 _ x5 : x x x x x x x x x x x x x : x2
x x x x x x x x x
6x3 4 2x2 : 6 X x X x x x : 3x
’ 2 x x x x
10x5 % 5x : 2x4
Remember, only factors may be cancelled
x2 % 2x x x x
: 2
2 x x
2 is not a factor of x
NIX
Examele 17:
Study the following carefully, giving the correct answers to
these that are marked wrong (X). Are any of the others wrong?
(a) x5 % x4 : x (b) a4 % 4a : 1 (x)
(c) 03 t O z 02 (d) 202 % 2a : 2 (x)
4 . _ i 3 53
(e) x 7 4x - 4x or 4 (f) x2y _ xy2 : l (X)
(9) ab % ba : O (X) (h) 5x5 _ 3x3 : Egg
2
(i) x3y2 % 6xy : 5-x or %x2y (j) x2y % 2X : EX

The answers to (b), (d), (f) and (9) should be 9%
(or %03), a, 5 and l. The others are correct.
QUESTIONS FOR PRACTICE:
34. Simplify:
(i) 30 x 30 x 30 (ii) 8x3 e 2x2
(iii) 30b x 2bc (iv) 2y3 x 3y2
(v) 1203b2c e 6abc (vi) 3x2y % ya
(vii) 2a x b2 e ab (viii) xzy % yx2
(ix) 3X X 2X2 X X3 (X) 3x2 x 3x2 % 3x3
Just as division is the reverse of multiplication, so the extracting of a root is the reverse of raising to a power.
Since 33 : 3 x 3 x 3 27, then i/E7 : 3
25 : 2 x 2 x 2 x 2 x 2 : 32, then %/52 z 2
You must distinguish between 3 V; where 3 is a factor mulitiplying the square root of x, and 3 V; , where the 3 indicates that the cube root is requ Thus 5 V; is five
’red.
times the square root of x, whereas g V; is the fifth root of x.
Do not confuse powers with factors:- x8 - x x x x x, 3x x or x e x s x. The figure before the x which multiplies it is also called a coefficient. Thus in 3x, 3 is the coefficient of x.
QUESTIONS FOR PRACTICE:
35. Write down:-
(a) The cube of 2x, 302 and abs.
(b) The square of 2x, 302 0nd abs.

10.
ll.
12.
l3.
l4.
l5.
16.
INTRODUCTION TO ALGEBRA. PAGE 15.
(c) A square root of x4, 9x2 and 1602b6.
(d) A cube root of 803, 27a9 and 12503b6.
(e) A fifth roof of x5, 015 and 32blo.
-------- oOo--------
ANSWERS TO PRACTICE QUESTIONS
. x
l OOOn m; I66 m; 12y months, 1000 cents.
70 m; 7t m; (This is similar to Example 7).
(0) 60C or R.60; (b) 5n cents or R EC’
x
20 kg,_ 2 kg,
1% A b years old; 181; z or 1% A %5 years old
x km- 22 km’ (’ e 2 x b)
y ’ 6O ’ 1’ " 60 A
. 9h
gh cents, R 100’
a A a : 20.
a A b : b A a.
Rx : 100x cents.
a b
bx;:l.
x
x cents : R100.
ax
a per cent of x - 160’
The sum of the first n odd numbers is n2.
1
- ofa:l.
O
a A a A a : 3 x a or 30.

________________________.______________
17. p months : f2 years.
18. n x i : l.
n
19. a x a
ll
0
21. vG';"E : a
22. 3J:5_: a
23. 2 x n or 2n is an even number.
24. 3 times a 1 o : 4 times a or 3a 1 a : 4a.
25. (i) 3, (ii) 3, (iii) 2, (iv) o, (v) 3, (vi) 54,
(vii) 7,5; (viii) 1,
26. (i) 3, (ii) 8, (111) 15, (iv) 1, (v) 2, (vi) 1,
(vii) 32, (viii) 10.
27. (1) 9, (11) 9, (iii) 243, (iv) 9, (v) 3, (vi) 6,
(vii) 6, (viii) 2.
28. 4o, 5b2, Zab, 17x.
29. 12 x 11, 12x, 12x,5 x 13, 50, 802, 24, 12x, l4xy, 50b.
30. 6x, 3x 1 3y, 3x, 2x - 2y, 7xy - 2x 1 y (no simpler form),
5x2 1 5x - 5.
31. 0, 3a 1 b - 3, x'1 y 1 xy (no simpler form), 3x 1 3y, 3x 1 6,
4a 1 6b 1 5, 2y _ 2, 2gb - 3bc, 4a 1 3b, y2 1 222.
32. 6x5 1 5x4 1 4x3 1 3x2 1 2x 1 1, 654 321.
4x4 1 3x3 1 2x2 1 x, 43 210.

4x5 - 3x4   2x3 - x2, 371 900.
x4   x3   x2   x - 1, 11 109.
33. a3   2a2   2a   1, b4   b2   4b   2, c2   c, 2d - d2,
2
2x2   3x - 5, 6y3, 7x2   x   2, x2   x   l, x   xy   2x,
x3 -3xy   2y.
34. 27a3, 4x, 6ab2c, 6y5, 202b, 25% , 2b, 1, 6x6, 3x
35. 8x3, 2706, a3b9, 4x2, 9a4, a2z6, x2, 3x, 40b3, 20, 3a3, 50b2,
x, a3, 2b2.
-------- oOo--------
RRC 4944

BRACKETS.
A0 MULTIPLICATION AND DIVISION:
In Arithmetic, anything which is enclosed in brackets is to be worked first - the order being Brackets, Multiplication, Division, Addition and Subtraction, working from left to right. This applies also to Algebra, but in some cases the process within the brackets cannot be performed unless the values of the letters are known, and we have to use a different method of simplifying.
Consider the following expressions:- (1) 2 x 3 # 5, (ii) 2 x (3 e 5), (iii) 2x e y, (iv) 2(x t y). If these are stated in words, we have - Multiply 3 by 2 and add 5 for (i). Similarly, (iii) means ’multiply x by 2 and add y’. On the other hand, for (ii) we must first add 3 to 5 and then multiply the result by 2; similarly in (iv) the sum of x and y is to be doubled.
Notice that the numerical results of (i) and (ii) are quite different (ll, 16) but that (ii) is the same as adding twice 3 to twice 5. In the same way if we add 10 x 3 to 10 x 7 we get 10 x (34-7)orlOO;-9xll-!-9xl3:9(ll-Pl3).
This is the same for all numerical cases and may be generalised as ax e ay : a(x t y). So that although a(x t y) means multiply the sum of x and y by a, we can work it out without knowing the values of a, x, y.
This can also be shown by a diagram.
The area of the whole figure is equal to the sum of the areas of the parts:a(x e y) : ax e ay.
By reversing the process we get the rule for division,
a(x # y)
x # y
since ax e ay
(ax e ay) % a

01’ (IX 4’ Oz
:xe
a Y
If ax is the number n and ay the number m, this becomes
m
(mhrntm : 2i-
0 O O 0
Choose different values for n, m, a and work out a few such sums.

B. ADDITION AND SUBTRACTION:

2.

ADDING BRACKETS:

Compare 3 t 5 e 7 and 3 e (5 t 7)
ll-o-7-20ndlli-(7-2).
The results are the same, and this is true for all such additions. The sum of x, y and z is the some as the result of adding x to the sum of y and z.
e.g. x t (y e z) : x # y e z and x e (y - z) : x t y - 2
In fact a t sign in front of a bracket has no effect on the bracket; not so for subtraction.

SUBTRACTING BRACKETS:

Compare (i) 10 - (3 t 5) with (ii) 10 - 3 e 5. The first means from 10 take the sum of 3 and 5, the second means from 10 take 3, then add 5. The difference in the two expressions is that in (i) both the 3 and the 5 are to be subtracted, not the 3 only - it may be worded ’from 10 take away 5 more than 3’ - in (ii) only the 3 is subtracted and the 5 is added.
Consider this problem:-
The total weight of 3 packages is 17 k9,, if two are known to weigh 5 kg. and 9 kg. respectively, what is the weight of the third? Answer 3kg. This could be obtained in two ways - either add 5 and 9 together and take from the whole - or take each package away separately.

A man has 20 coins, 7 are 10c pieces, 4 are lc pieces; how many are neither 10c pieces nor lc pieces? Answer 9. This can be worked as 7 e 4 are lOc pieces and 1c pieces.
Therefore 20 "-(7 e 4) are neither.
Or, as 20 - 7 are not 10c pieces, i.e., 13.
13 - 4 are neither 10c pieces nor lc pieces.
This is perfectly general rule and may be written thus
a - (b e c) : a - b - c.
In words: if the sum of b and c is taken from a, the result is the same as if b and c are both taken from 0.
Consider a - (b - c). This means that the difference of b and c has to be taken from a (or b less c from o). If we take b we take too much and must put something back; how much?
vagrq 7&7 .. _. ._,
A man earns a Rand and pays b Rand tax; this is c Rahd-too much "" tax. What tax should he have paid? How much has he left?
He should have paid (b - c) Rand tax
He will have left a - (b - c) Rand
If he has already paid b Rand tax, he will have only (a - b) Rand.
When the excess is refunded he will have c - b e c Rand.
Take a numerical example: 17 - (9 - 2). If you take 9 from 17 you have taken too much - and must put back 2, therefore
17 - (9 - 2) : 17 - 9 e 2
5 - (4 - 1) : 5 - 4 s l : 2
This also is a general rule:-
a - (b - c) : a - b e c
Work other numerical examples till you are sure of the two ’ rules for subtracting brackets.

BRACKETS. PAGE 4.

SUW%ARY:

We have now given you four general rules for addition and subtraction of brackets. They must be completely understood, so we are repeating them again:-

x e (y t z) : x t y t Z
X t (Y - z) : x t y - z
a - (b e c) : a - b - c
a - (b - c) : a - b e c

These four rules for addition and subtraction can be stated as follows (and you must memorise them):-

If a bracket has a e sign before it, when the bracket is removed the sign of each term is unchanged.

If a bracket has a - sign before it, when the bracket is removed the sign of each term is changed (t becomes - and - becomes t).

Do not change the sign unless you remove the bracket. If the bracket has a number or letter immediately in front of it this rule of signs is not affected, but the rule for multiplication applies also - everything within the bracket must be multiplied by the term outside the bracket, e.g.

N
V
II
x t 2(y e x t 2y t 22
H
x e 2(y - z) x # 2y - 22
x - o(y e z)
x - 0y - oz
x - a(y - z) : x - ay e 02

The rules for addition and subtraction must be adhered to when brackets are inserted, thus:-

x e 5y e 52 becomes x e 5(y e z)
a t 2b - 26 becomes a t 2(b - c)
but x - 5y t 52 becomes x - 5(y - z)
and o - 2b - 2c becomes a - 2(b t c)

Be very careful about this.
Minus in front of a bracket changes the sign inside, but not till the bracket is removed.
Never write a - b(c e d) : a (be - bd) (Wrong)
This should be a - b(c t d) : a - bc - bd
Remember that each term has its own sign, and if you change the order of the terms be careful that each one has the correct sign.
QUESTIONS FOR PRACTICE:
1. Remove the brackets from the following, and collect terms when possible:-
(i) x - (y 5a 32) (ii) 3x - (x - 3)
(iii) x t 3(y # x) (iv) 4x e 2(x - y)
(v) a(b - c) - b(c - a)(vi) a(a e b # 2) t 2(a e b - 2)
.(vii) (y 1x z)x t y 4- xz (viii) (x3 e 3::2 4- 2x) e x
(ix) 02 t b2 t 2ab - a(b - a) e b(a - b)
(x) (14x2y _ 7xy2) % 7y
2. Use brackets for the following:-
(a) A man buys x articles at p cents and y articles at 9 cents each; find his change in 10c pieces from a one Rand note.
(b) In a class of m students, n students are away but k new ones come; how many will be present?
(c) Think of a number x, subtract y, and double the result, again subtract y and double the result.
(d) From x student subtract half the difference of y and 2.

C. MORE COMPLEX BRACKETS:
It is sometimes necessary to have more than one set of brackets.
In such cases, the inner brackets are removed or simplified first,
e.g.
3 Eu - 2(b - c)j : 3 fa - 2b   2cj : 30 - 6b   6c
2(0 _ Eb - (c - d)J) : 2(a - Eb - c   d1) : 20 - 2b   2c - 2d
Instead of a bracket 0 bar is sometimes placed above an expression;
this has exactly the same meaning as a bracket, thus
2(x   ; - i) : 2(x   y - 2), etc., and
2(x - y   z) : 2(x - y - 2), etc.
Simplify afb   (c - J_I"E)J aEb   c - d - e1
ab   ac - ad - ae
QUESTIONS FOR PRACTICE:
3. Simplify (i) 3x2 - 2Ex - (x2 - 2X)J
(ii)   3(x - y) _ 2(x   y)J   xy
(iii) 1 # xEl   x(l   xl   X)J
4. Remove brackets from the following and simplify the result
where possible:-
(i) 3(2x - 1)   4(1 _ x)
(ii) x(x2 _ 1)   2(x - 3)
(iii) 20(x - y)   3b(y - x)
(iv) a(a - b)   b(b - a)
(V) 5(x2 - x - 1) - x(x - 5)
(vi) 30(0 - 3) - 4b(b - 4)

(vii) 4(x - 7) - 7(x - 4)
(viii) 7 (3k - 5n ; 5m) - 6k(3 - n - m)
(iX) x(y - 2) - 2(x - y) ; y(z - X)
(x) 2(x - y - 2) - 3(y - z - X)
5. Fill in the missing terms in the following:-
(i) 30 - 3b - 2x - 2y : ( ) - 2 ( )
(ii) 02 ; 0x2 - bx2 ; b : a( ) - b( )
: a( ) ; b( )
(iii) x2 ; xy ; xz - yz : x( ) ; Z( )
: X( ) ; y( )
6. Simplify (i) (403 # 402 - 80) % 40
(ii) (02 - ax - ay) % a
(iii) (10a3 - 1502y) % 50
(iv) (8a2b2 ; 60b) % 2ab
(v) (02 - 20) % a
-------- oOo--------
ANSWERS TO PRACTICE QUESTIONS.
1. x - y - 32; 2x ; 3; 4x ; 3y; 6xl- 2y;
2
20b - ac - bc; a
x2 ; 3x ; 2; 2a2 ; 20b; 2x2 - xy.
; ab ; 4a ; 2b - 4; xy ; 2x2 ; y;
2. 10 - 25-1-31 10c pieces; m - (n - k); 2E2(X - Y) - Y) i

3. 5x2 - 6x; x   5y   xy; 1   x   x2   x3   x4.
4. 2x   l; x3   x - 6; 20x - 20y   3by - 3bx;
02 -r b2 - 20b; 4x2 - 5’
I
302 - 4b2 - 90 J, 16b; -3x;
3k - 35n   35m   6nk   6km; 2yz - 22x; 52 - x - 5y.
5. 3(0 - b) - 2(x   Y); a(a   x2) - b(x2 - 1);
0(0   x2) Jr b(1 - x2); x(x ’0’ y) ,L z(x - y);
x(x   z)   y(x - Y).
6. 02   a - 2; a - x - y; 202 - 30y; 40b   3; a - 2.

FORMULAE

A. i LITERAL PROBLEMS:

A formula is a general statement, using letters, which includes all particular numerical examples.

When faced with a more difficult literal problem, always invent for yourself a similar numerical problem; work this first and then perform the parallel working with letters instead of numbers.

Remember that the letters stand merely for numbers not for quantities, and therefore the unit (cents, feet, etc.)must be expressed. Never say 'The speed of the train is x', 'The cost is 3',etc. Say, 'The speed is x k.p.h.', and 'The cost is 3 cents'.

It is sometimes necessary to distinguish between even and odd numbers. Since all even numbers are multiples of 2, such a number may be represented by 2n; by giving n the values 1, 2, 3, etc., consecutively, we obtain the even numbers 2, 4, 6, etc.

2n, 2n 0 2, 2n # 4, 2n - 2 will represent even numbers whereas 2n s 1, 2n - 3, etc., will be always odd numbers.

Examele 1:

If a train is observed to travel x metres in t seconds, what is its speed in k.p.h.?

In t seconds it travels x metres.

x x 60 x 60

Hence in 60 x 60 seconds it travels _____;_____ metres.

. . . x 392! l

i.e., in 1 hour it travels t x 1555 km

. . 18

. . Speed is 3-? k.p.h.

Examele 2:

If a men do a piece of work in x days, how long will it take b men to do the same piece of work?

(Use numbers, for example - if 15 men take 3 days to do the work how long will 10 men take?

FORMULAE. PAGE 2.
1f 15 men take 3 days then 10 men take 2 $015
days
N.B. 10 men take longer.)
If a men take x days
Then b men take x x E days
. ax
Tlmc taken : - days
b
Examele 3:
If the hot tap fills a both in x minutes, and the cold tap in
y minutes, how long will it take to fill if they are turned
on together?
(Use numbers, for example _ hot tap takes 17 minutes
cold tap takes 13 minutes.
1 l
In 1 min. hot tap fills 17 of both, cold top 15 of bath
. . . l 1
Therefore, 1n 1 mxn. both taps f111 17 $ 13 of both
13 1 17
: 13 x 17 of both
If I7_g9I5 of bath takes 1 min. to fill,
Then %; : 13 of bath takes 1Z_%612 mins.)
Hot tap takes x mins.; cold tap takes y mins.
In 1 min. hot tap fills % of the both
In 1 min. cold tap fills % of the both
In 1 min. they together fill % 1 i : X;$_5 of bath
1-2;5 of bath takes 1 min.
Therefore 51 takes __5X__ mins.
Xy X41y
It takes __EX__ mins. to fill.
x y y

QUESTBDNS FOR PRACTICE:
l. A man travels at x k.p.h. How far will he walk in (i) 0 hours, (ii) b minutes? (iii) How long will it take him to walk c km?
2. If the exchange rate is x francs to the Rand, express (i) Ry in francs, (ii) 2 francs in Rand.
3. If x men can harvest a crop of potatoes in y days, how long will m men take?
4. If Income Tax is Xc. in the Rand, what gross income will yield Ry net income? What will be the net income from a gross income of R2?
5. A room is x m, long and y m, wide. If it is z m, high what is the area of the four walls? If there is a carpet a m. by b m. in the room, what is the area of the border round it?
6. 8 kilometres are approximately equal to 5 miles. Find a formula for converting miles to kilometres.
7. Find a formula to express in cents per gm. 0 price given in Rand per kg.
B. SUBSTITUTION:
We will now consider problems in which numbers are substituted for letters in a formula, with the exception of one letter. We have to find the unknown value of this letter.
Before substituting in a formula, see that all your quantities are in the correct units.
Study the following examples very carefully:-
Examgle 4:
If s : ut t %ft2, find 5 when u - 18, f : 32 and t : 2.
s : 18 x 2 t % x 32 x 22
: 36 e 64
s : 100

Examele 5:
L . . .
If the formula T : 213/; glves the t1me T secs. for a swlng
of a pendulum of length L metres, find the time of a swing of
o g m pendulum, if g : 10 and7r : 2; .
22 5
T 2 2 x 7 x 8.10
22 l
: 2 x 7 x VIE;
_2x2_2x1__1_1
_ 7 4 _ 7
Time : 1; secs. or 1,57 secs. to 2 decimal places.
Examgle 6:
. . PRT .
In the s1mple Lnterest formula I : $66, R 15 the rate per cent
per annum and T the time. Find the simple interest on R375 at
4 per cent for 216 days.
216
T255
. I _ 375 x 4 gig
' ' 100 x 365
QUESTIONS FOR PRACTICE:
8. The formula for the volume of a sphere is V : % w-ra where r is
the radius. 22
Find the volume of a sphere of diameter 7 cm if zr : -7 If the
surface area is given by S : 41F r2, find the area of the above
Sphere.
. . r n
9. The harmula for compound 1nterest lsI : P Ell t 100) - i1
where RP is the principal, I per cent the rate and n the number
of years. Find the compound interest on R400 at 5 per cent for
4 years.
10. To convert temperature readings from Fahrenheit to Centigrade,
subtract 32 and multiply by five-ninths. Find a formula to
express this. What is the Centigrade reading of (i) 400 Fahren-
heit? (ii) 2120 Fahrenheit?

EpRMULAE.s PAGE 5.

C. lTHE NEED FOR UNITSzl

There are one or two mistakes in connection with formulae which occur very frequently. Some of these are given in the following examples. Correct them.

(a) If 1 kg. of tea costs x, then 7 kg. cost 7x.
(b) If the length of a rectangle is 0 cm. and its width is b cm., its area is ab cm.
(c) Let the price of sugar be x cents.
(d) If the speed of a train is x and the time is y, the distance is xy.

The correct form of these is given below:-

(a) If 1 kg. of tea costs x cents, 7 kg. cost 7x cents.
(b) Its area is ab sguare centimetres.
(c) Let the price of 1 kg. of sugar be x cents, or let the price of sugar be x cents per kg.
(d) If the speed is x k.E.h. and the time is y hours, the distance is xy kilometres.

ANSWERS TO PRACTICE QUESTIONS.

1. Speed : x k.p.h. (i) In 0 hours he goes ex km.
.. . bx
(11) In b minutes he goes 36 km.
(iii) He walks c km. in & hours.
2. R1 : francs. (i) Ry
xy francs.
(ii) 2 francs : RE

FORMAL PAGE 6.
x men take y days hi man takes xy dast
Therefore m men take 51 days
N.B. 1 man takes longer than x men.
Tax on R1 is Xc therefore R1 yields R(l - I66) net income
100 - x . .
R(-I66-) is net income from R1
Ry is the net income from R(-J$XhL-O
100 x x
Rz yields R(ZEIOO _ XJ)
100
Area of 4 walls is 22 (x t y) m2.
Area of floor is xy m2 : area of carpet is ab m2.
Therefore area of border is (xy - ob) m2.
8 km. : 5 miles, therefore, x miles : g? km.
Therefore K- -2x gives the formula; x gives the miles and K
the kilometres.
Let RP be the cost of 1 kg.
Then 1 000 9. cost P x 100 cents
P.100 P
1000 : IBi cents
l 9. costs
Hence p ; $6 is the formula, where P is the price in Rand per
kg. and p is the price in cents per g.
V . 5-wsr3. r : 3,5 1r : 22
3 11 7
V I 22 x2 7 7 _ 11 x 49 _ 222
"3 2H??t 3 _ 3
Volume : 17:% cm3.
5:47rr 3,,572L;
r I :
S : ng?;154
Surface area _ -l54 cm2

In
9. I:P((l-PibB)-l) (P-400,r-5,n_4)
I : 4oo((1,o5)4 _ 1) Now 1,052
1,1025
1,10254 1,216 approx.
Hence I : 400 (0,216) : 86,4
Answer : R86 to the nearest R.
10. If C0 is the Centigrade reading corresponding to F0 Fahrenheit
Then c : g(r - 32)
40, c : 2(40 - 32) z 59
(i) When F 9
II
The reading is 4,40C.
(ii) when F : 212, c : g(zlz _ 32) : g x 180
The reading is 1000C.

WWI .
'rHlJJIF' .
.N .
vm'i (I
M4:
ll 1';
m .'
q .
t!ufarjcf:;r
.3(
1p
I
un'

DIRECTED NUMBERS.
A. NEGATIVE QUANTITIES:
You will realise that in Algebra there are two classes of numbers, positive numbers and negative numbers.
If we begin from O and continually add on one, we have the ordinary arithmetical numbers:-
1, 2, 3, 4, 5, and so on.
These numbers are called positive numbers, and we can, if we like, prefix the sign e to each of them, thus
#1, 4-2, 4-3, e4, 4-5,
Similarly, if we start with O, and continually subtract one, we have a different series of numbers, called negative numbers, and disting-uished by the sign:.
Thus -1, -2, -3, -4, -5, ...
The negative sign must never be omitted. Observe that it is usual to omit the positive sign in Arithmetic.
The important point to note is that the negative sign shows what kind of number we are dealing with. It does not merely mean 'subtrcct!
If we write down -9, the negative sign shows us that here is a negative number 9. But, we are not directed to subtract 9 from any other quantity.
Here is another method of explaining negative numbers. Take the case of a thermometer in the Centigrade Scale. On this scale, freezing point is indicated by zero, or O; boiling point by 100. Should the temperature fall below freezing point, say 9 degrees, we should denote this temperature by '-9'.
-90 means 9 degrees below zero.
A rise of 30 from this -90 will bring the temperature to -60.
Therefore, -90 t 30 : -60. (If we add 3 to -9, we get -6).
A fall of 30 from this same -90 brings us, on the scale, to inO.
0
Hence -90 -3 : -l:o

Again, beginning frog -90, a rise 3f 120 brings us to #30.
Thus, -90 e 120 : e3 , or simply 3 .
Now, suppose we start from 120, and the temperature falls 150.
In this case, 4-12o - 150 : -30.
It is very useful practice to work out several of these examples from an actual thermometer, and prove the results in algebraic form. If you have no thermometer, draw one as on the following page. Alternatively you can, of course, draw the line horizontally, as shown.
0n the horizontal scale, count forward from 0 for your positive numbers and backwards for your negative numbers.
Start at e10.
Add e5. This brings you to e15.
Now add -15. This means "count backwards’ 15.
We are at zero.
Therefore e10 e 5 e (-15) : 0 (notice the bracket).
Any negative number added to a positive number of the same size cancels it, giving the result ’0’.
Thus: e9 e (-9) : 0. (Count 9 ’foruard’ from 0, and then count 9 in the reverse direction).
Practise several of these ’countings’, and express your steps in the proper form, using all signs.

M
30
25
4-20
445
4-10
- 5 Use this graduated line as your thermometer, and make up your
10 own examples.
-25 -20 -15 -10 -5 O 5 10 15 20 25

B. ADDITION AND SUBTRACTION:

1. ADDITION:

Consider the following problem:-

A lift starts on the 5th floor, it goes Up 3 floors, down 2 floors, down 4 floors, up 1 floor, up 2 floors, down 5 floors. Where is it now?

He may use directed numbers for this, taking t to mean up and - to mean down. The result of the various movements is obtained by adding them.
Thus we have
e5 4, (e3) -r (-2) 4- (-4) tr (4-1) i- (4-2) 4- (-5).
We need to know how to add a negative number.
Take a simple example: (t3) e (-2).
This would mean start on the 3rd floor and go down 2.
We know the result will be the lst floor or t1.
Therefore (t3) t (-2) : t1.
This is the same as 3 - 2.
Suppose it were (-3) t (-2). This would mean start 3 floors down - third basement floor - and go down 2 more floors, this would bring us 5 floors down.
(-3) t (-2) : -5.
Thus the result of adding a negative number is to reduce the given quantity by that number, or counting on the vertical number scale, to go down.
We have already seen in considering brackets that a t before a bracket does not alter the sign within the bracket and you will notice that this rule applies with directed numbers.
When adding with directed numbers take the sum of all the Eggatiyg_numbers from the sum of all the Bositive numbers.
If the sum of the negative numbers is greater than that of the positive numbers, the subtraction must be performed the other way and the result will be negative. Consider (e10) - (e17). If you take th from e10 the result is 0.
If you now take a further t7 you get to -7. Therefore (e10) - (tl7) is the same as - E(tl7) - (th)J.

SUBTRACTION:
This brings us to the rules for subtractioni
There are two important points to remember:-
(a) If you take a larger number from a smaller one the result will be negative, e.g., l6 - 20 : -4. If you take 16 from 16 nothing is left, we require to take away 4 more, therefore, 16 - 20 : -4.
(b) Subtraction can be made to depend on addition. The question what is 7 - 3 can be worded "What must be added to 3 to get 7?"Then, if you move to the right on the horizontal number scale the answer is plus; if you move to the left the answer is minus.
Examgles:
l. (i) What must be added to 2 to give 7
(ii) what is 7 - 2? (5)
2. -(i) What must be added to 7 to give 2? 7
(ii) what is 2 - 7? (-5)
3. (i) What must be added to (-2) to give (e7)?
(ii) What is (s7) - (-2)? . (s9)
4. (i) What must be added to (s7) to give (-2)?
(ii) what is (-2) - (#7)? . (-9)
5. (i) What must be added to (#2) to give (-7)?
(ii) What is (-7) - (#2)? (-9)
6. (i) What must be added to (-7) to give (e2)
(ii) what is (s2) - (-7)? 4 (s9)
7. (i) What must be added to (-2) to give (i7)?
(ii) What is (-7) - (-2)? (-5)
8. (i) What must be added to-(-7) to give (-2)?
(ii) what is (-2) - (-7)? (s5)

Go over these very carefully and see that you agree with the answers. It is essential that you understand the principle before trying to learn a rule. Draw your own scale, unless you can imagine it in your head.

-4 -3 -2 -l 0 l 2 3 4 5

A subtraction sum asks the question ’How far is it from the second number to the first’? Your answer must give the direction also, e to the right, - to the left.

Thus (e5) - (-3) means how far from (-3) to (e5)? Answer, 8 to the right, i.e., t8.

(-5) - (-3) means how for from (-3) to (-5)? Answer, 2 to the left, i.e., -2.

Can you see the rule for yourself? It is the same as from brackets ’- changes the sign’.

Thus - (-3) is the same as e3

- (e3) is the same as -3.

QUESTIhNS FOR PRACTICE:

1.

P.)

;1) (e7) e (e2) (ii) (e7) e (-2) (iii) (_7) t (e2)

(iv) (-7) e (-2)

(e) (e3) t (ell) (ii) (t3) t(-11) (iii) (-3) e (e11)

(iv) (-3) e (-11)

(i) (e7) - (e2) (ii) (e7) - (-2) (iii) (-7) - (e2)

(iv) (-7) - (-2)

(i) (e3) - (e11) (ii) (e3) - (-11) (iii) (-3) - (e11)

(iV) (-3) - (-ll)

5. (i) o - (t5) (ii) 0 _ (-5) (iii) 0 e (-4)
(iv) (t1) t (-1)

C. MULTIPLICATION AhD DIVISION:

This rule is the same for both and depends on the known fact that
a x b : b x 0.
Consider 3 x (-2).
We know that 3x means x t x e x whatever value x has.
Therefore, 3 x (-2) means (-2) t (-2) t (-2) : -6.
Now (-2) x (3) has no meaning, since you cannot take 3 for -2
times, but (-2) x (t3) must be equal to (t3) x (-2) which is
The rule is that if we multiply together two numbers of
oegosite signs the answer will be minus.
To find a meaning for (-a) x (-b) we can consider the
following problem:-
A train is travelling due south at 60 k.p.h. We take this
as a speed of -60, where t60 would be a speed going north.
We take distances north of a certain town as e and south
of this town as -. If the train passes through this town
at noon, time before noon is negative (-) and after noon
is positive (- .
Where was the train 2 hours before noon? Distance is
found from the product of speed and time. The question is
how far does a train travel in -2 hours at -60 k.p.h.
Answer (-2) x (-60). We know that 2 hours before noon,
the train was 120 km north of the town.
Therefore, (-2) x (-60)
ll
e120.
This is a particular example, but the general statement is
true also (-a) x (-b) : tab.
In words: If two numbers are both negative their Broduct is
eositive.

________.________________
Naturally, if both numbers are positive the product is positive,
so the rules may be expressed thus:-
The product of two numbers of LIKE SIGNS is POSITIVE
The product of two numbers of UNLIKE SIGNS is NEGATIVE.
If ab : c, then a : E
(-3) x (-2) : -6, therefore, (-6) % ()2) : -3
and (-6) % (-3) :)2
(-3) x (-2) :)6, therefore, ()6)-% (-3) : -2
We know that (-6) % ()3) : -2
This gives the same rules for division.
The quotient of two numbers of LIKE SIGNS is POSITIVE.
The quotient of two numbers of UNLIKE SIGNS is NEGATIVE.
QUESTIONS FOR PRACTICE:
6. (i) (-4) x (-3) (ii) (-4) x (-3) (iii) (-4) x (-3)
(iV) (-4) x (-3) (V) (-4) x 0
NJ
(i) (-12) x (-5) (ii) (-12) x (-5) (iii) (-5) x (-12)
(iv) 0 x (-5) (v) (-1)2
8. (i) (-24) - (-5) (ii) (-24) % (-6) (iii) (-24) - (-6)
(iv) (-24) - (-6) (ii) (-6) - 1
9. (i) (-15) - (-3) (ii) (-15) - (-3) (iii) (-15) - (-3)
(iv) (-15) - (-3) (V) 3 % (-l)
10. (i) (4)3 (ii) (-2) x (-3) x (-4)
(iii) (-5) % (-%) (iv) (4) - (-8) (v) 0 e-(-2)

D. SUWWARY:

These rules for addition,_subtraction, multiplication and division can be generalised thus:-

a e (sh) : a s b
a4-(-b) : a-b
a x (-b) : - ab
(-a)xb :eab
(-a) x (-b) : sob

Note the following:-

(a)
(b)
(e)
(d)
(e)
(f)
(g)
(h)
(i)
(k)

If any number
If any.number
If any number
If any number
If any number
Since a x a :
is
is
is
is
is
a2
a - (eh) : a - b
a - (-b) : a s b
a _ (-b) : - b
(-0) i b : - E
(-a) % (-b) 4%
multiplied by O the product is Oi
multiplied by 1 it is unchanged.
divided by 1 it is unchanged.
multiplied by -1 its sign only is changed.
divided by -1 its sign only is changed.
and (-o) x (-o) : 02, either a or no is a
square root of a 2.
We shall take V43 to mean the positive square root of 02.
All squares are positive.
Even powers of -l are 1: odd powers of -l are n1.
Only like terms may be collected in addition and subtraction.
If a bracket is preceded by a sign,
fhe sign of each term is changed.-
If a bracket is multi
bracket is removed
that expression.
, each term must be multiplied or divi&ed by
when the bracket is removed,
plied or divided by on expression, when the

Exameles:
Find (i) The sum, (ii) The difference of the following pairs of expressions:-
9. 3x 1 2y, 4x - 3y 10. 2x 1 15y, 13x 1 7y
ll. 5xy - 3y2, 7x2 1 4yx 12. 8x - 5y 1 2, 4x- 3y - 22
13. x2y 1 2y2, y2 - 2xy 14. 3x2 - 2x 1 1, 3 - 2x - x2
9. (1) 3x 1 2y (ii) 3x 1 2y
4x - 3x 4x - 3x
7x - y -x 1 5x
10. (i) 2x 1 15y (ii) 2x 1 15y
13x 1 7y 13x 1 7x
15x 1 22! -11x 1 8x
11. (i) 2 5xy - 3y2 (ii) 5xy - 3y2 N.B. xy : yx
7x 1 4xx 7x2 1 4xx
7x2 1 9xx - 322 -7x2 1 xx _ 3V2
12. (i) 8x - 5y 1 2 (ii) 8x - 5y 1 2
4x - 3x - 22 4x - 3x - 22
12x - 8y - 2 4x - 2x 1 32
13. (i) x2y 4- 2y2 (ii) X2)! 1 2Y2
xzx 4. 322 - 2xy xzy 1 y2 1 2Xy
14. (1) 3x2 - 2x 1 1 (11) 3x2 - 2x 1 1
-x _ 2x 1 3 -x2 - 2x 1 3
2x2 - 4x 1 4 4x2 - 2
QUESTIONS FOR PRACTICE:
11. Add (i) 6a and -0; (ii) -3b and b; (iii) -c and 0,
(iv) -2d and -5d; (v) -e and e; (vi) -f and -f.

_____________________________.
12. Subtract (i) 60 from -0; (ii) -3b from b,
(iii) -c from 0, (iv) -2d from -5d,
(v) e from -e (vi) -3f from -3f.
13. Multiply (i) 60 by -0, (ii) -3b by c,
(iii) -c by 0, (iv) -2d by -5,
(v) e2 by -e, (vi) bl by -xy.
14. Divide (i) 662 by -0, (ii) -3b2 by 3,
(iii) 0 by -c, (iv) -20d3 by -5d,
(v) 2e2 by -2e, (vi) -4x2 by -4x.
l5.Ifx:l,y:-l,z:0,a:3,b:-3,c-.-2,findthe
value of:-
(i) a b x, (ii) a - x, (iii) b b y,
V(iv) b - y, (v) c - 2, (vi) a b 2y
(vii) 20 - y, (viii) a(x - y) (ix) b(y b x)
(x) 2(02 - b2) (xi) abc (xii) x # y (a _ b)
... bc , . 0c x2 b 2
__ ._ _____1_
(X111) 0 (XIV) b (XV) Ob
. a2 b x2 2 2 2 b 2 "
(XVI) ---- (xvii) a - b b c (xviii) x - y1 b 22
b2 y y2
(xix) H (xx) 0%;
l6. Simplify:
(i)(x-y4-2)4r(x4by-2)4-(x-y-2)
(ii) (02 - ab - b2) b (ab - 02 - b2) b (b2 - a2 - ab)
(iii) (x2 - 2x - 3) - (3x2 4 2x - 1)
(vi) a(ab _ 02) - b(ab - b2)
(v) 0(02 - a b l) - 3 (02 b a - 2)

(vi) (3x - 2y) - (2x 1 3y)

(vii) (x - 3y) 1 (2y - 3x)

-------- oOo--------

ANSWERS TO PRACTICE QUESTIONS.

1. 9, 5, -5, -9 2. l4, -8, 8, -14
3. 5, 9, -9, -5 4. -8, 14, -l4, 8
5. -5, 5, -4, O 6. 12, -12, -12, 12, O
7. 60, -60, -60, O, 1 8. 4, -4, 4, -4, -6
9. 5, -5, 5, -5, -3 10. -l, -24, 10, -%, 0
11. 5a, -2b, -c, -7d, 0,-2f, 12. -7a, 4b, c, -3d, -2e, 0
13- -6o2,13bc, 0, 10d, -e3, xy 14. -6a, -b2, 0, 4d2, -e, x
15. 4, 2, -4, -2, ’2: ll 7! 6r 6: OI 18! 5/ 2! 2! "5;: ll 4,
O, 0, -18
16. 3x - y - z, -02 - ob - b2 or -(a2 1 ab 1 b2),
-2x2 - 4x - 2 or -(2x2 1 4x 1 2), 02b - a3 - ab2 1 b3,
3 2
0-40-20167 X-5y, -2X"y

EQUATIONS.
A. SIMPLE EQUATIONS:
A statement of equality between two expressions is called an equation, e.g., a certain number multiplied by 3 is 6; this can be written as 3 times (the required number) : 6. From which we can say the required number : 6 e 3 or 2.
To save writing ’the required number’, which is sometimes called the ’unknown’, we find it convenient to let a letter stand for the number. Let x be the required number, and then the statement becomes 3 x x : 6 or 3x : 6.
We have already said that letters stand for numbers, but 22: for numbers of things. It is most important to remember this, as in dealing with problems you will meet with statements connecting quantities as well as those dealing simply with numbers, e.g., a box and its contents together have a mass of 1,25 kg. and the mass of the contents is four times as much as the box, find an equation from these facts.
We can say ’mass of box e 4 times mass of box : 1,25 kg; If we mass of the box be x kg. this becomes
x kg. t 4x kg. : 1,25 kg.
From which it is clear that x e 4x : 1,25 which is the required equation connecting the numbers.
Examgles:
Here are a few simple statements which you can easily write as equations for practice.
1. A certain number is added to 4 and the result is 20.
2. A certain number is multiplied by 4 and the result is 20.
3. If 4 is taken from a certain number the result is 5.
4. If a certain number is halved the result is l.
5. If a certain number is taken from 7 the result is 3.
6. A number exceeds 4 by 9.

10.

EQUATIONS. PAGE 2.

Think of a number, double it and add 3, the result is 13.
Five times a certain number less 4 is 11.
The denominator of a fraction is 5 more than the numerator
and the fraction equals %.
The sum of 3 consecutive numbers is 18.

Notes on ExamBles l - 10:

The equations are:-
x t 4 : 20; 4x : 20; x - 4 : 5; %x : l;
7 - x : 3; x - 4 : 9; 2x # 3 : 13; 5x - 4 : 11;
x e 5 : 2x; 3x t 3 : 18 (where x is the number).

If a fraction is equal to one-half, then the denominator must be
twice the numerator.

Have you remembered that if x is the first of a series of
consecutive numbers, the others are x e 1, x t 2, etc.?

ExamEles:

When you are sure of the above equations, find an equation
crmawcting numbers from the following statements.

11
ii.
13.
14,
15.

A bottle and a cork cost 10c and the bottle costs 6c more than
the cork.
R4 worth of coins consists of 200 pieces and 10: pieces and
there are thirteen more 10c pieces than 20c pieces.
20 kg. of tea at 60c 0 kg. are mixed with a quantity of tea
at 70C 0 kg. The mixture is worth 66 cents 0 kg.
A reEtangle is twice as long as it is wide and its area is
16cm .
Three years ago a man was twice as old as his son. Their
combined ages are now 72.

Notes on Examgles ll - 15:
You must remember always to state what x (or the letter you choose) represents and give the correct unit.
11.
12.
13.
14.
15.
Let the cork cost x cents. (Not ’let the cork cost x’,
x t x t 6 : 10 nor ’let the cost be x’).
Let x be the number of 20 pieces (Not let x be 20c pieces).
20x t 10 (x t 13) : 400 or 30x t 130 : 400.
Let x be the number of kg. at 70c.
Total quantity is (20 e x) kg. and total cost is 20 x 60 t 70 X cents.
1200 t 70x : 66(20 e x)
Let the width be x cm.
2x x x z 16 or 2x2 : 16.
Let x years he son’s age now; three years ago he was (x w 3) years.
Father’s age is 72 - x years now; he was (72 m x - 3) years.
Therefore 69 - x : 2(x _ 3).
B. SOLUTION 0F EQUATIONS:
Look again at the first ten equations formed above. You can probably guess the answers to those where they are not obvious at once, e.g., if
(i) 4 t a number is 20, the number is 16.
(ii) 4 times a number is 20, the number is 5, etc.
In order to solve more difficult equations you must fully understand the method of working these simple ones.

To get 16 in (i) you take the 4 from 20.
To get 5 in (ii) you divide the 20 by 4.
In other words, you work the question backwards, for example,
(a) 5 is added to a number to make 7; take away the 5; then the number is 2.
(b) 3 is taken from a number and leaves 9; add the 5 on again; then the number is 12.
(c) A number is multiplied by 4, and the product is 24; divide by 4, then the number is 6.
(d) A number is divided by 3, and the quotient is 11; multiply by 3, then the number is 33.
The imoortont thing to remember in solving equations, is that the two sides stait 9323i, and must be heat eguol,
The eguation is like a balance;.if you add a quantity to one side you must add the some or on equal quantity to the other.
If you take a quantity From one side, you must take the some or on equal quantity from the other.
If you multiply, or divide, one side by a qUantity, you must multiply or divide the other by the some or on equal quantity
These rules depend upon four axioms:-
(o) If two numbers are equal and we add equal numbers to them the results are equal.
(b) If two numbers are equal and we take equal numbers from them the results are equal.
(c) If two numbers are equal and we multiply them by equal numbers the results are equal.
(d) If two numbers are equal and we divide them by equal numbers the results are equal.
Remember you must treat both sides of the equation alike.

EQUATIONS. PAGE 5.
Study the following worked examples carefully. The written statements in square brackets are to explain the method and should be omitted as soon as you are sure of the work.

Examele 16:

3x e 7 : l9

ETake 7 from both sides) 5

1

X.

3x 12

EDivide both sides by 31

x : 4

(Check the answer by putting x 4 into equation. Then 3x t 7 :
12 1 7 : 19)

Examgle 17:

%(X-2):5

EMultiply both sides by 31

x-2:15

EAdd 2 to both sidesj

x : 17

1 1

(Check: 3(x - 2) : 5(15)

: 5

N.B. In working back to find the unknown the steps are reversed and taken in the reverse order.
%(x - 2) means 'take 2 from x, and divide the result by 3'.
We first multiply by 3, on? then add 2. If you added 2 first the equation would become 3(x - 2) e 2 z 7, which is no help in solving.

Examele 18:

%X-5:2

tAdd 5 to both sides)

741:7

IDivide both sides by %J

x : 7 x g (To divide by a fraction, invert and

x : 9% multiply)

(Check %x - 5 3 % x 2% _ 5

z 7 _ 5 : 2)

$929342-

5 - 2x : l

fAdd 2x to both sidesj

5 ; l   2x

Eque 1 from both sidesj

4 2 2x

01v1de both sides by 2

x : 2

(Check 5 - 2x : 5 - 4 : l)

A shorter method of solving this is as follows:-

5 - 2x : 1

(Take 5 from both sidesj

-2x : -4

EDivide both sides by uZJ

x : 2
pus. (-4) 4- (-2) : 21
Examgle 20:
5x - 3 : 2x e 9
EHere we must collect terms containing x on the left and numerical terms on the right)
ETake 2x from both sidesJ
3x - 3 : 9
EAdd 3 to both sides)
3x 12
ll
iDivide both sides by 33
x : 4
(Check: L.H.S. 2O - 3 : 17
ll
R.H.S. : 8 4- 9 : 17)
You should now be able to work several examples omitting the writfen statements, and in examples such as 19 and 20 above, two steps may be performed at once, i.e., add 2x to both sides and take 1 from both sides, and take 2x from both sides and add 3 to both sides.
Compare examples l9 and 20 above with the following:-
Examele 21: Examgle 22:
14 w 3x : 2 14x - 7 : 5x e 2
12 : 3x 9x : 9
x 2 4 x : l
QUESTIONS FOR PRACTICE:
Work the following questions. These include some where x stands for a negative number. If necessary revise addition, subtraction, multi-plication and division of negative numbers.

1. 4x-327 2. 3x-4-2:ll
3. 25-3x:l3 4. 9x4-5zl5-x
5. 5x31227 e. 4-3x:l6
7. 2x-5z3x-4 8. 5(x-2):25
2
9. %x-2:%x4,1 10. -3-(x-l)-rZ:6
C. FRACTIONAL EQUATION$:'
If an equation contains several fractional terms, it is best to clear it of fractions by multiplying both sides by the lowest common multiple (L.C.M.) of the denominators.
x - 9 x l
48 8 :Ezti
4 x 88 - ll(x - 9) : 4 x x - 44 (L.C.M. of 8, 22, 2 is 88)
352 - 11x t 99 : 4x - 44
352 3 99 s 44 : 15x
x _ 1252
_ 15
x z 33
EQIE: There is no need to collect terms in x on the left always; it is better to keep the x terms positive as in this example even if they are then on the right. Always work the equations down the page, keeping the equal signs underneath each other. Do not write .'. before each line - it is not necessary.
QU_ESTIONS FOR PRACTICE:
ll. - : - 3 l - -; 2 3 x
12 _ ,__ _ _
3 4 12 3x 5 - 4 t 2
x - l x - 2 _ x t l x t 2
13 3 ' 4 ' 2 i 3
11,.le xgls4:o 15 3-5:1-9-
X X

D. PROBLEMS:

We have already seen how a_statement connecting different quantities can be made into an equation and problems should not prove difficult if the following points are remembered:-

(0) State clearly what number the letters stand for, and give the correct unit.

(b) Bring all your quantities to the same unit.

(c) First write the equation connecting quantities, then the numerical equation. Later, when you are sure of the work, only the numerical equation should be written.

(d) Always answer the question. The answer to your equation will be x : a number. Put the number into your definition of x to get the answer to the given problem.

(e) Check your answer in the given problem, not in your equation. You may have made a mistake in forming the equation.

Study carefully the following examples. Statements in square brackets show the method, but need not be written as part of the solution.

Exomele 23:

The sum of R2,25 is made up of 10c and 5c pieces and there are 27 coins. How many 5c pieces are there?

Let x be the number of 5c pieces.

Then 27 - x is the number of 10c pieces.

(Their value in 5c pieces is 2(27 - x) 5c pieces)

(The value of R2,25 is 225 cents or 45, 5c pieces)

ix e 2(27 - x) 5c pieces : 45 5c pieces)

x t 2(27 - x) 45

x e 54 - 2x 45

9 : x
There are 9 5c pieces
(Check: 9 x 5c s 18 x 10c : 45c I l80c : 225C or R2,25)
Examele 24:
A man is 25 years older than his son. In 3 years’ time he will be twice as old as his son is now. How old is he?
Let x years be man’s age now; in 3 years he will be (x e 3) years
Son’s age is (x - 25) years.
ETherefore (x i 3) years : 2(x - 25) years)
x i 3 : 2(x - 25)
x s 3 : 2x - 50, 53 : x. The man is 53 years old.
(Check: son is 28 years. In 3 years man will 56)
Examele 25
In a test 2 marks are given for every sum right, and 1 mark is deducted for every sum wrong. If a boy works 9 sums and obtains 9 marks, how many sums were right?.
Let x be the number of sums right; 9 - x are wrong.
He gains 2x marks and loses 9 - x marks.
ETherefore 2x - (9 - x) marks : 9 marksJ
.’. 2x - (9 - x) : 9
2x I x - 9 : 9
3x : 18
x : 6
He had 6 sums right.
(Check: 12 marks gained, 3 marks lost).

QUESTIONS FOR PRACTICE:
16. A garage has a number of cars and motorcycles. If there are 13 vehicles altogether and they have 0 total of 40 wheels, how many cars are thete?
17. Two boys have R3,60 between them, one has 011 10c pieces and the other all 20c peices; if there are twice as many 10 pieces as 20c pieces how much money has each?
18. My average cycling speed is 6 k.p.h. faster than my walking speed. If 1 cycle for 3 hours and walk for half an hour, and cover 33% km., at what speed do I walk?
19. If a train travels at 40 k.p.h. it takes 15 minutes longer to complete its journey than when its speed is 45 k.p.h. How long will the journey take at 50 k.p.h.
20. A man bought a number of articles for R22,50. He sold 70 per cent of them at a profit of 15c each and the rest were thrown away; if he lost Rl,50 on the deal, how many articles did he buy?
E. LITERAL EQUATIONS:
In the previous equations we have been concerned with obtaining numerical values. Sometimes we have to solve equations in which the value of the unknown is found in terms of other letters which occur in the equations. These are called literal eguotions. Solving them is done in exactly the same way as for numerical equations.
ExamEle 26:
Find x in terms of a and b in the following equations:-
(i) 6x - a : 2x # b (ii) %5 _ 4 : g _ 6
(i) 6x - a : 2x e b
gSubtract 2x from both sides, and add a to both sides. This brings t e terms in x to the same side)
6x - 2x : b e a
4x : b e a
x:b-l-a
A

(ii) 925-4:%_6
(Eliminate fractions by multiplying both sides by L.C.M., i.e., 61
30x - 24 2b - 36
2b - 36 t 24
30x
30x : 2b - 12
Divide both sides by 30
2b - 12
30
X :
QUESTIONS FOR PRACTICE:
Solve the following equations for x:-
21. 8x - b : 3x t 4p
22. px t 6q : bx - 2q
2x fx 2f
23. 3 - f _ 5- - 5-
x x
24. h_5h:7_x
F. FORMULAE:
Useful examples of literal equations are provided in the changing of the subject of a formula. This is a most important subject for you to know.
Exomele 27:
To make I the subject of the formula
T : ZIP, for the time of swing of a pendulum
277E, Divide by 2 11’
1f - J; Square
.4
ll
Nl-l
l

_..._.._
2
I73: 51 Multiply byg-
4 A
I _ 12.9
4r
Excmgle 28:
Make d the subject of S : g(Zc A n n 1d)
% X 5 : 2a A (n n l)d
2% - 20 : (n - l)d
23 ; 20h : (n _ l)d
2(S - on)
d : h(n - 15
Questions of both 0? these types should present no difficulty if
you have already mastered the methods of solving equations.
QUESTIONS FOR PRACTICE:
25. Solve for x:
(i) 3x - 2a : 4a A x
(ii) 1 - x : x - __i__
a - b a A b
(iii) (x - o)2 A (x - b)2 : (x A c)2 A (x A d)2.
. x - a x - b x - c
_____
(1v) A I A
:0
26. If A : 21r (r A h) find h in terms of r and A where?T : 2%
(opprox.) What is the value of h when A : 154, r : 2?
ut A %gt2, find 9 in terms of S, u, t. What is 9 when
27. 3
S 64,U:O,t:2?
II II

28.
29.
10.
ll.
13.
14.
EQUATIONS. PAGE g.
% 2 % : %, find U in terms of v, f. What is u when v : 5,
3
_ _7
f _ 34. 2
If y : l 2 35; , find x in terms of y.
3 2 x L
-------- oOo--------
ANSWERS TO PRACTICE QUESTIONS.
x:2% 2 X:3 3 x:4
x ; l 5 x : -l 6 x : -4
x : -l 8 x : 7 9 x : -12
x s 7
5x 2x 3x . 2 3 x
13:721- 7 12. 3x-gzz25
5x : 8x 2 12 - 9x 60x - 8 : l5 2 10x
6x : 12 50x : 23
23
x z 2 x : 0,46 or 36
x - l x - 2 x 2 l x 2 2
3
4(x - l) - 3(x - 2)
6(x 2 l) 2 4(x 22)
4x - 4 - 3x 2 6 6x 2 6 2 4x 2 8
x 2 2 : 10x 2 14
-12 : 9X
x : 21%
x 2 l x - l_2 4 : 0
425
5(x 21) 2 4(x - l) 2 20 x 4
5x 2 5 2 4x - 4 2 80
II I!
O O

9x : -81
-9
X
15. 3 _ 5 : 1 _ 3
X x
(Multiply both sides by x1
3x - 4 : x - 3
2x 1
1
x : 7
16. Let x be the number of cars: there are 13 - x cycles
ETotal number of wheels isJ 4x e 2(13 - x) : 40
4x e 26 - 2x : 40
2x : 14
x : 7
There are 7 cars
(Check 7 cars have 28 wheels
g cycles i2 "
1_3 Q "
17. Let x be the number of 20c pieces; value : 20x cents
2x is the number of lOc pieces; value : 20x cents
e. 20x e 20x : 360 fin cents)
40x 360
x : 9
The boys have Rl,80 each.

EQUATIONS. PAGE 16.
18. Let x k.p.h. be walking speed, x i 6 k.p.h. is cycling speed
EIn 3 hours 1 cycle 3(x i 6) km. ; in half an hour I walk
%x km.J
. ’ 3
. 3(x i 6) i 7x : 33;
3x i 18 i %x : 33%
3
3%x : 15;
x : 4%
I walk at 4% k.p.h.
(Check: 1 cycle 3 x 10% or 31% km., I walk % x 4% km. : 2-
km.)
19. Let x hours be the time for the journey at 45 k.p.h.
Distance is 45x km..
At 40 k.p.h. the time taken is (x i %) hours
’. Distance : 40 (x i %) km.
45x : 40(x i %)
45x : 40x i 10
5x : 10
x : 2. Distance is 45 x 2 or 90 km.
90 km. at 50 k.p.h. takes %% hrs. or 1 hr. 48 mins.
(Check 90 km. at 40 k.p.h. takes 2 hrs. 15 mins.
at 45 k.p.h. takes 2 hrs.)
20. Let x be the number of articles. Each costs 2:50 cents
Each sells for (2250 i l5)cents
X
70 per cent sell for Z x x (2250
10 x
i 15) cents : 2100 cents

._.;1::.1 15) : 2100
7 x 225 5 10,5x : 2100
10,5x : 2100 - 1575
525
x:l_O,-5-:5O
He bought 50 articles
(Check: 1 article costs 2:30 cents : 45 cents
S.P. of %Oof 50 articles : 35 x 60 cents : 2100 cents (or R21)
Loss on whole deal was Rl,50)
21. 8x - 3x : 4p 5 b
5X:4p’i-b
- X : 4E g b
22. px 5 6q : bx - 2q
px - bx : - 2q - 6q
x(p-b): -8q
Ha
p - b
Alternatively,
6q 5 2q : bx - px
8q : x(b - p)
x : __321__
b-p
2x fx 2f
23. 5"-f:-a-5-
EMultiply all through 31

EQUATIONS. PAGE 18.

2x - 3f : fx - 2f

2x - fx : 3f - 2f

X(Z-f): f

_ __i__

X _ 2 _ f

X X

24’ F - 5h 2 7 ’ x

EMultiply by L.C.M., i.e., 3h)

3x - x : 7 x 3h-x x 3h

2x : 21h - 3hx

2x 2 3hx : 21h

x(2 2 3h) : 21h

_ 21h

X ’ 2 2 3h

25. (i) x : 30

.. 1 1

(11) E2x : a - b   o 2 b1

1

x : -2---2-

2(02 _ b2)

(iii) x2 terms go out.

2x(o 2 b 2 c 2 d) : 02 2 b2 - c2 - d2

02 2 b2 - c2 - d2

2(a 2 b 2 c 2 d)

X :

(iv) 3obc

bc 2 co 2 ob

26.
27.
28.
29.
RRC 4948
EQUATIONS.
27,-(r-frh)
rmlltlb
u n
H
When
(I)
I
C
d’
I l
Nl-I
O
r!-
M

5 x 15
40 - l5
3
Collect x2 terms and take the square root of x2:
x2(3 - y)
2
3y - l
3 1
Y
X
3
f
____-.___

SIMPLE GRAPHS.

A. NATURE OF GRAPHS

l. GRAPHICAL REPRESENTATION OF FIGURES:

A graph is a method of representing figures pictorially. There are several different types of graph but the one most frequently used in elementary economic analysis is the ordinary two-dimensional graph. This simply means that our picture is intended to represent two related sets of figures. Sets of figures are related if a change - in one set involves a change in the other set.

For example, suppose we weigh all the boys in a school. We are likely to find that boys gain in weight as they grow older. We could divide the boys into age groups, say ll - 12, 13 - l4, l5 - l6, l7 - 18. For simplicity we can assume that none are under 11 or over 18. Suppose we found that the average weight of boys in each group was as follows:-

Age Group	11 - 12	13 - l4	l5 - l6	l7 - 18
Average Mass (kilos)	46	54	61	67

These figures can be shown on squared or ordinary graph paper as in Figure l.

.H mmDon
AVERAGE MASS KILOGRAMS

AXES:

Notice that the horizontal border of the graph is labelled with an X in addition to the age groups and the vertical border is labelled with a Y in addition to the mass in kilos. These borders are called axes, i.e., we have an X axis and a Y axis.

You might now ask why we have shown the ages on the X axis and the masses on the Y axis. We have noted that the average mass depends on the age, i.e. the mass changes with the age. The mass of the boys is thus a variable that is dependent on another variable, which we can call the indeeendent variable - in this case the age. By mathematical convention, the independent variable is plotted on the X axis and the dependent variable on the Y axis.

The scale, or number of squares chosen to represent each unit, is a matter of choice, e.g. on our vertical axis each small square represents one kilo. (A break is indicated on both axes, as all the values Up from zero are not shown.)

Notice how each point on the graph is plotted. The normal rule is to move to the appropriate point on the X axis and then move Up parallel to the Y axis, to a position facing the appropriate point on that scale. This process for the 13 - 14 age group, with average mass of 54 kilos, is indicated by the dotted lines. The points plotted for each pair of figures are then joined to form a continuous line.

POSITIVE AND NEGATIVE DIRECTIONS:

You might also ask why we moved to the right and upwards from the corner of the graph. Why not to the left and down? Again we plead mathematical custom. The corner really represents a centre point for a full graph. This centre point is called the Boint of origin (from which all points on the graph start) and is given the symbol 0.

All points above 0 represent values greater than zero for the dependent variable; all points below 0 represent values below zero, i.e. negative values. Similarly, all points to the right of 0 represent values greater than zero for the independent variable and all those to the left are less than zero. The term "axis" now becomes clearer. It is a centre line with all positive numbers (greater than zero) on one side and all negative numbers (less than zero) on the other.

If we return to our imaginary group of schoolboys, we could rearrange our figures so that they fall on both sides around a centre point representing the average age and average mass of all the boys. The X axis now represents years above and below this average (which, say, is 15 years 0 months) and the Y axis represents kilos above and below the average mass (which is, say, 60 kilos). We may now produce the following table:-

Yeurs above (t)
or below (-)
the average -4 -3 m2 -1 0 e1 t2 e3 #4
Kilos above (e)
or below (-)
the average -l8 -l3 -8 -4 0 t3 t6 t8 th

These figures are plotted on the graph in Figufe 2.

.
- _
.l. m _ _
g
.
41L; .1111 .140 1.. 1 .LI. t .1 L1V1r1_ lzv .1 . . V
... $1.... V:.111.1111T1_1:.11n11. 1 1.11. U17. ..111U1w. 1.1 - U11 11:
1.1911 11 .11.. .. 1V1 . .111 V - 101 H41 1 AA.
1 . ELIKJVL? Ier1T1WT11L111111 1L 1.11V..l1..1 1!. 11111.1... 1l.1VJH_._ 1111.11.
. 1. IL L 11.10119 Jlfh
I .
mm ..1.. .1. 11.11 1.11. .1 1111-1 "Hal . ._ 11.
. 1;. 1 1. 1. .111 .11 I11 11. 1.1111 1.1.1-1 111.1 .
Wm"? U1... - 11 : 1.111 11.1. 111 1.11% 11m1 11.1.1"! .11 .111 1.1111 L1.
1 1 1 1111!. A 0. 161.1; AAV1A1 frat
_ 1
1 THL: I L ..L 0 . V . .
V %%I1HI’ 1.1.11.1. nLIrA . 1 1.148191L111 . . 1
.111 . 111.1 V 1111 . . 1 . ..
. . my. 111.111.
4 . 1 1111 Fr... .. .111 .1111
m 1.. WI- 1 1 1 .
11f .
1.11
11.1 1.1.311

Here is another example of a graph containing both positive and negative numbers. Suppose a study of a group of students indicated that the average student studied for 30 hours a week and obtained an average grade of 55%; the study also revealed that those who studied longer than the average obtained higher grades and those who studied less obtained worse than average grades. The following figures were recorded:

Hours of Stud Number of Marks
above (t and above t and
below (- the below (- the
average average
-5 -l7
-4 -12
-3 - 8
-2 - 5
-l - 3
average 0 , average 0
H 4- 4
4-2 4, 7
e3 4- 9
t4 e10
4-5 ell

This suggests that grades are dependent upon hours of study. When we graph the figures, therefore, the grades are the dependent variable and are plotted on the Y axis. The hours of study are the independent variable and are plotted on the axis. The resulting graph is shown in Figure 3.

uvt..xvrv
91 : .V.t
PACE 7.

8. SOME USES OF GRAPHS

l. VISUAL EFFECT:

One use of a graph is to create effect. The relationship between the two sets of figures is seen much more easily from a graph than from a table, especially by people unused to working with figures. This is one of the most common uses of graphs but in many ways the least important. It is also open to misuse. Notice the apparent difference between the graphs of Figures 4a and 4b. The line in 40 seems much steeper than in 4b. A closer look, however, reveals that this is simply because the scale of the X axis is different (the scale in Figure 4b is four times that in Figure 4a). In fact the lines are really the same - in each case they join the points x : o, y : o and x : 8, y : 4.

The term "curve" is used to describe the line joining the points on a graph, even when this is a straight line. Strictly, such a line is called a "linear curve".

FIGURE 4b
rf. fTrt .
h JV: MIA.
.vl 1.x?
.T. 11
L 4
A .11 ?HI...
11.4 IiT
LLL:
A71.
2104
7m
.1! J
. 11
1.. 4t
TILL
Ur A
tww. .

2.

SIMPLE GRAPHS. PAGE 10.

INTERPOLATION:

A more important use of a graph is to assist in the estimation of values of x and y lying between two known points.

Consider the following graph which represents the production of beer in the Country X between 1990 - 1996 (Figure 5). The production for 1993 has been deliberately left out. From the curve, however, we can estimate that production in that year was around 34,5 million barrels. (In fact the coirect figure is 34,7; which is not far from our estimate).

FIGURE 5.
1993
-994
1995
1996
1997
PAGE 11.

This use of a graph is called interpolation. We have to assume, of course, that the figures do follow a regular trend and that nothing happened in the interval to upset that trend. Failure to take into account probable changes can lead to some bad errors.

Consider the following graph of spirit consumption in Country 2 for the years 1997 to 2000 (Figure 6). If we assumed that consumption continued to rise between 1998 and 2000 we would estimate a figure of 34,5 million litres for 1999. In fact the true figure was 31,6 an unexpected fall. The explanation for this is likely to lie in tax increases and incomes controls.

A graph, therefore, should never divert attention from an intelligent study of all the facts surrounding any set of figures.

EXTRAPOLATION:

Even more dangerous is an attempt to predict figures from 0 Projection of a curve into the future on the basis of past experience, without fully taking into account all likely influences.

Figure 7 is a graph showing the population of the Country Q between 1981 and 1998. The figures for 1981 - 1986 are almost a straight line. Anyone in 1986, asked to predict the probable size of the population in 1998 could be forgiven for offering a figure of about 58,4 million. In fact, however, the curve started to become less steep in 1987, but between 1990 and 1993 the line was straight again. A revised prediction for 1998 made in 1993 might have been 56,7 million. In fact, the population stopped rising in 1993 and started to Fall slightly, so that the true figure For 1998 was 55 835 000. Few people foresaw such a change in the early 1980’s and plans were then made for education services based on expected increases which did not take place. These plans have proved to be very costly for many people.

Notice here again that the Y axis is broken (below 52,5 million). This enables us to concentrate on values between 52 and 55 million without wasting space on the graph.

POPULATION IN MILLION
-vM. 7
,mwmuw
on .u .u .. .
uM
H
wowerHH.oz H2
Hemp:
Boom.
1.1mm
14w
WM
.
WHH
LMH oJA
WW
14 4 1
mm.
o.
ITTr
iw
.Jvr Li
t(5 11
r177 7.7V.
aLlo. 14 6:71.
,VrW , HM- IATT
.o ... a .-
l 1
1:.
7%
94.. 6.1M4
.4 1.1
1
:3
27.41 ,. ,7 1:7 v
YW.
Iva.
Tot 3
Hocxm V.
’95 96
92 973 9 4

C. STRAIGHT-LINE GRAPHS

1. RELATIONSHIP BETWEEN X AN) Y:

_______.____________________

In spite of the dangers of making predictions by extrapoation, this remains an important use of graphs. The danger is reduced, of course, if we know, or can with reasonable safety assume, that the curve has a definite and predictable shape. If the curve is regular then there must be a definite relationship between the Figures on the Y axis and the X axis. Remember that we normally expect the Y figures to be dependent upon the X figures, so that such a regular relationship is not an unreasonable expectation.

Suppose we take the simplest possible case where y (standing for the figures on the Y axis) always equals x (standing for the figures on the X axis). Then we have a straight line passing through the point of origin (0) of the graph (as in Figure 8).

’FIGURE 8.
311; 1..-
54 4:1 I
PAGE 17

If, as in this illustration, the scales on both axes are the same, the line (or curve) bisects the angle between the two axes and is thus at 450 to each axis.
This relationship of 1:1 is unlikely to occur very often.
More frequently we find that y is either some fraction or some multiple of x. If we know this fraction or multiple (called a coefficient) then we can find the value of y for any given value of x. We can also, of course, find a value of x for any given value of y.
Suppose y is g5. When x : 1, then y : 1% . When x : 5,
l . . . .
y : 7E and so on. leen thls constant relat10nsh1p, we can read off from the graph (Figure 9) the values for y given by the various possible values of x. When x is 6, for instance, we can see that y is 9. When x : -2 then y is -3.

FIGURE 9.
H H; 1,1.4 H
iyLll
GRAPH 0F Y
PAGE 19

__________________________._
Thus we can express any linear (straight-line) curve which passes through the point of origin (0) by the general equation:

y : bx

when b simply represents the value of the coefficient.
We have already seen, however, that not all curves are likely to pass through the point of origin where both y and x : O. In Figure 10 we see that when x : 0, y : 3 and in addition to this the coefficient is 2. Thus, when
5
we increase x by l we also increase y by g. If we reduce x by 1 then we reduce y by 3.
The equation for this curve then is y : 3 e gx. If we use the letter "a" for the value of y when x : 0, and "b" for the coefficient, then we arrive at a general equation for a stroight-line graph:

y:a#bx

.2 $5on
Van
WHT
v r

CURVES WITH NEGATIVE SLOPES:

So far we have examined situations where y rises or falls as x rises or falls. It is possible for y to fall when x rises, i.e. to move in the opposite direction to x. Suppose instead of the equation y : 3 t g x we had y : 3 g5 , then the resulting graph would appear as in Figure 11. Notice that when x : 5, y : 0, when x - 10, y : - 3 and when x : -5, y : 6. Check with the equation that these values are correct.

nHOCmm E. .

When the values of y move in the opposite direction to those of x we can say that there is a negative relationship between the two. In strict mathematical terms, the general equation for the straight line graph remains y : a t bx, but the value of the coefficient, b, is negative. In the above example, b : -%, signifying that the value of y has to fall by g of any change in the value of x.

GRADIENT:

The term "gradient" refers to the steepness of the slope of the curve. We have seen earlier, however, that this can appear to be altered by the relative scales of the X and Y axes. We need, therefore, to have a more precise method of measuring gradient.

Geographers use "gradient" as a measure of the vertical distance in relation to the horizontal distance between two points. Thus, a gradient of l in 10 means that a slope either rises or falls one metre for every 10 metres of horizontal length. This is illustrated in Figure 12.

HMZAOm.
H n
28% S. zmzmm

Using Y and X to denote the vertical and horizontal movements as in our normal graphs, we can see that y : 1 when x : 10. If the slope remains the same, then y will equal 2 when x : 20. y is always one tenth of x, i.e.

-l_x
V "10'

The coefficient b in the equation y : o t bx is the some as the gradient. It measures the slope of the curve and is not affected by the relative scales of the axes on the graph.

2

A curve with a coefficient or gradient of I6 will slope twice as steeply as one with a coefficient of %6. Graphs will illustrate this Erovided that the Iglotionshie between Y and X scales is keet the same.

The gradient is on important element in the curve because it indicates the strength of the reaction in the value of y following any change in the value of x.

D. NON-LINEAR CURVES

1. SOME SIMPLE RELATIONSHIPS:

_________._________________

Not all graphs are straight lines. It is quite possible for the y values to have a regular relationship with the x values and the result to be something other than a straight line or linear curve.

Suppose, for instance, we have the relationship y : x2, then the values of x and y are as follows:e

Value of x: l 2 3 4 5

and so on.

Value of y: l 4 9 16 25

The resulting graph is shown in Figure 13.

t)om NV.
mHOCmm pm.

Notice that no value of y in this curve can ever be negative (Can you see why?)
Another interesting curve is produced by the equation
1
y 2 so that when x : l, y : l; x : 2,
;
l l
y _ 2; X : 3/ Y : 3!
and so on. The relationship for a range of positive values of x is shown in Figure 14.

Notice that if y : % then xy : 1. If for l we substitute any other fixed quantity and represent this by the letter "0" then the graph of y : g will have the some general shape. If we have a succession of different values for a, we could then have a series of curves all having a similar shape but at different distances from the point of origin. The table of values represented by the graph of Figure 15 is set out below:-

,	Value of x	Value of x
	a : 2	a : 3
1	2	3
2	1	3/2 (1,5)
3	2/3 (0,67)	1
4	2/4 (0, 5)	3/4 (0,75)
5	2/5 (0,4)	3/5 (0,6)
6	2/6 (0,33)	3/6 (0,5)
7	2/7 (0,286)	3/7 (0,43)
8	2/8 (0,25)	3/8 (0,375)
9	2/9 (0, 22)	3/9 (0,33)
10	2/10 (0, 2)	3/10 (0,3)

FIGURE 15.
y...JA_4A4. .
all). 1. V0
PAGE 31
2 AND
G
:
3 (POSITIVE VALU
E
50F x)

Test your understanding of this graph by calculating the values of y when x : % (a : 2) and when x : % (a : 3).
These examples are of well known relationships. Economists are often faced with sections of graphs which appear to have regular shapes. Their problem then is to estimate equations to fit these shapes, in order to make predictions for a range of values for the variables they are studying. When they do this they have to remember that they are also making assumptions about influences on these relationships, which may or may not turn out to be correct.
GRADIENTS:
It is an essential feature of the linear curve or straight-line graph that the gradient stays the same throughout its length. This is implicit in the equation y : u # hx, where the coefficient b retains the some value for all values of y and x. No such assumption is possible for non-linear curves, where the steepness changes as we move along the curve. Thus we cannot refer to the gradient of the curve, only to the gradient at a particular part of the curve.
Consider the curve represented in Figure 16. Note that as x changes from 4 to 6, y rises from 2 to 3. Thus there is a rise of ly in 2x. Further along the curve, however, a change in x from 7 to 8 produces a rise of 2y (from 4 to 6); here, then, there is a rise of 2y in lx. The slope has become much steeper.

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FIGURE 16.

Clearly the gradient is changing as we move along the curve. How then do we know the gradient at any particular point on the curve? Let us look more closely at just part of the curve, e.g. that represented by the movement from 7 to 8 on the X axis and from 4 to 6 on the Y axis. This part is shown in Figure 17.

FIGURE 17.
PAGE 35

If we join the points x : 7 and y : 6 we get a straight line with a gradient of 2 in 1. The coefficient of x to produce a change of ly is thus 2 over this particular section of the curve (x changes by l and y changes by 2). Such a section of a curve is usually called an arc.
Now suppose we shorten the distance of this arc by moving the line to the right until it just touches the curve at A. A line that just touches in this way is called a tangent. The tangent has the same slope or gradient as our original line and we can, therefore, say that this gradient of 2 applies to that Boint of the curve which is identified by the letter A. The tangent measures the steepness of the curve at A, i.e. the rate at that point at which the value of y is changing in relation to x.
It is difficult to obtain an accurate measurement by drawing a tangent but it is often sufficient to be able to find a rough estimate of the steepness and direction of the slope. In practice, if we know the equation of a curve we can apply the mathematical technique of differential calculus to find out the gradient, or rate of change, at any point along that curve.
LORENZ CURVE:
A long curve is a graph, where both the x and y axes are expressed as a percentage, e.g.,
100
WAGES
0 % 100
EMPLOYEES

ond may be a straight iine (l) or a curve (2)w Note this graph will always start at O and finish at lOOL

REVISION QUESTIONS:

1.

6.

Draw the graph of y z 3 t gi-for values at x from -2 to 5. Use the graph to estimate the value of y when x is 4 and check your answer by calculating the value of y from the equation.

Draw the graph of y : -4 t %x for values of x from -3 to 6. What is the value of y when x : 0?

Explain what is meant by interpolation from a graph. ch does this differ from a projection?

Which is the coefficient of x in the following equation:

y:lO-i-’g-)E?

What would be the general shape and direction of the graph ’ produced by the equation?

y : 20 - g5 ?

Which of the following equations would you expect to produce a linear curve:

y : a t 2x2

y : a - 4x

y : a - x3 ?

above.
When x : 4 y
5% from the graph.
his also is the same as we calculated
y : 3
2x
3
for values of x : -2 to 5
QUESTION 1:
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PAGE 39.
SIMPLE GRAPHS.
QUESTION 2:
for values of x from -3 to 6.
a
I
i):
5
Graph of y : -4
-1 O 1 2 3 4 5 6
-2
x : -3
4.l7,v 411v;
u r v;
e r? _ 11
Tlf
-4   4x
GRAPH OF y :
From the graph it can be seen that when x : o y z -4.
QUESTION 3:
Can be answered by referring to the lecture.

QUESTION 4:
yle’ng
5
The coefficient of x is -
9
QUESTION 5:
The general shape would be a straight line sloping downwards to the righ1
QUESTION 6:
y : a - 4x would produce a linear curve.

YOUR TESTS!

Once you have completed the test contained in this set of lectures please post it to:

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