

VERY IMPORTANT QUESTION (11) ON  
 Binary Operation

FORM IV

MATHEMATICS-2

Time: Two hours

Answer ALL questions in Section A and FOUR questions in Section B.

All necessary working must be shown.

SECTION A

1.

$$A = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} R & 0 \\ 2 & 1 \end{pmatrix}.$$

Work out the matrix products  $A B$  and  $B A$ .

Given that  $A B = B A$ , find the value of  $R$ .

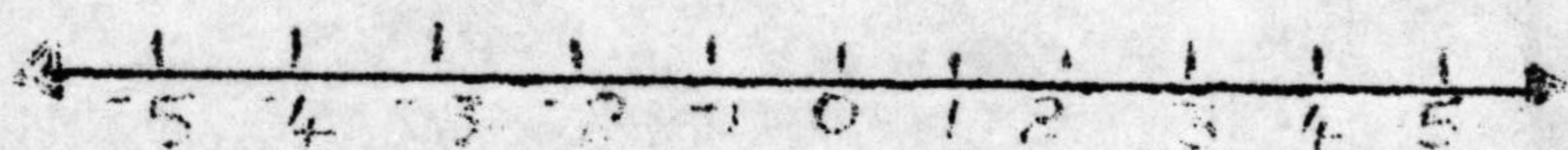
2.

(i) Solve the equation

$$\frac{X+2}{3} - \frac{2X-1}{4} = \frac{1}{2}$$

(ii) Given that  $ax + by + c = 0$ , find an expression for  $y$  in terms of  $a, b, c$ , and  $x$ .

3.



Make two copies of the above number line and indicate by thickening a part of each line, the solution sets of the inequalities.

(i)  $2x + 1 \leq -3$

(ii)  $-14 \leq 3x - 2$ .

Hence find the values of  $x$  in the set of integers which satisfy both these inequalities simultaneously.

4.

Show that  $X - 2$  is a factor of  $X^3 + 2X^2 - 5X - 6$  and hence write this expression as the product of three linear factors.

5.

Given that  $f: X \rightarrow 3X$  and  $g: X \rightarrow X - 4$ , write down, in the form  $X \rightarrow \dots$ , the inverse function  $f^{-1}$  and the composite function  $f^{-1}g$ . Find the value of  $X$  for which  $f^{-1}g(X) = f(X)$ .



## SECTION B.

6. Functions  $f$  and  $g$  are defined by  $f(X) = \frac{1}{X}$ ,  $g(X) = 4 - X$ .

(a) Using a scale of 4cm to 1 unit on each axis, plot and draw with the same axes the graphs of those two functions for

$$0,2 \leq X \leq 4,0.$$

(b) From your graphs, estimate the two values of  $X$  for which  $f(X) = g(X)$ .

Write down an equation of which these two values are the roots and simplify it.

7. A ball was thrown vertically upwards and, after  $t$  seconds, its height,  $h$  metres, above the ground was given by

$$h = 33 + 4t - 5t^2,$$

(a) Calculate the height from which the ball was thrown.

(b) Find the speed with which it was thrown.

(c) Find the time when the speed became zero.

(d) Calculate the greatest height above the ground reached by the ball.

(e) Find how many seconds elapsed from the time the ball was thrown until it reached ground level.

8. Make  $X$  the subject of the formula

$$y = \sqrt{\frac{(x^2 - 7)f}{p}}$$

(ii) Show that  $x-4$  is a factor of  $6x^3 - 23x^2 - 6x + 8$

and factorise this expression completely. Hence write down the solutions of the equation

$$6x^3 - 23x^2 - 6x + 8 = 0$$

(a) in the set of rational numbers,

(b) in the set of integers.

9. Using a scale of 2cm to represent 1 unit on the  $x$ -axis and 4cm to represent 1 unit on the  $y$ -axis, draw the graph of the function defined by  $f: X \rightarrow \frac{4}{X} + \frac{X}{4}$  for  $1 \leq X \leq 7$ .

(a) Draw the tangent to the graph at the point where  $X=2,5$  and find its gradient.

(b) Using the same scale and axes, draw the graph of the function defined by  $g: X \rightarrow -\frac{1}{3}x + 4$

(c) From your graph find the values of  $x$  for which  $f(x) = g(x)$ , and form, but do not simplify, an equation of which these values are the roots.



10.

The diagram shows a circular window in a church. Semi-circles are drawn on each side of a square ABCD such that the semi-circle touch the circumference of the outer circle at P, Q, R and S. Given that  $AB = 2x \text{ cm}$ , find, in terms of  $x$ , the area of the outer circle and the area of the shaded region. Show that the ratio of these areas is  $8\pi : (\pi - 2)$ .

Given that the area of the shaded portion is  $3000 \text{ cm}^2$ , calculate, to the nearest cm, the radius of the outer circle.

11. Given that  $S = (A, B, C, D)$  where

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

copy and complete the following table in which the operation \* is matrix multiplication.

*	A	B	C	D
A		C		
B				
C			D	
D				

For this operation write down

(a) the identity element in  $S$ ,

(b) the inverse of  $B$  in  $S$ ,

(c) the element of  $S$  which is equal to

(i)  $A^4$ , (ii)  $B^{10}$ , (iii) the inverse of  $C^{11}$ .

12.

(i) Show that  $(x+2)$  is a factor of  $2x^3 - x^2 - 13x - 6$  and factorize this expression completely. Hence solve the equation  $2x^3 - x^2 - 13x - 6 = 0$

(a) in the set of integers,

(b) in the set of rational numbers.

(ii) Solve the equation

$$x^2 - 10x + 1 = 0$$

and show the product of your answers equals 1.